Determination of the Contrast and Modulation Transfer Functions for High Resolution Imaging of Individual Atoms

by

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Abstract

This report describes a high resolution optical imaging system that is intended to contribute to the development of a system that will be used to resolve individual atoms in an optical lattice. The contrast transfer function (CTF) and the modulation transfer function (MTF) are computed by the means of two different techniques. The CTF is found by imaging individual square wave patterns of different spatial frequencies and determining their contrast. The MTF is found by using a slanted edge method. This method is realized by imaging a slanted silicon edge with thickness 0.53mm. The edge is projected along the correct angle to give the true edge spread function (ESF). The ESF is numerically differentiated to obtain the line spread function, and finally the fast fourier transform (FFT) is performed on the LSF to MTF, which is then normalized. Both of these methods allow for an estimation of the resolution of the imaging system.
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Chapter 1

Background

1.1 Point Spread Function and Resolution

When evaluating the performance of a given optical imaging system, it is essential to understand the response of the system to the object being imaged. The Point Spread Function (PSF), which describes the response of an imaging system to a point object, is a measure of the intensity distribution of light emitted by the point source. In the far-field approximation (Fraunhofer diffraction), for light passing through a circular aperture, the point spread function is given by

$$\text{PSF}(\rho) \propto \left[ \frac{J_1(\rho)}{\rho} \right]^2,$$

where $\rho$ is the radial distance and $J_1$ is the Bessel function of the first kind.

The resolution of an optical imaging system, defined as the minimum distance between two point objects such that they are distinguishable, is often determined by the Rayleigh Criterion. The Rayleigh criterion states that the minimum resolvable distance is given when the maximum of one of the PSF’s lies at the first minimum of the other PSF. This statement translates to the maximum of the sum of the two PSF’s being about 81% of the minimum for a rectangular aperture and about 74% for a circular aperture [1] (see Fig. 1.1). In this work we are dealing with circular apertures, and therefore use the 74% criteria.

The resolution is quantified by the following equation:

$$R = \frac{1.22\lambda}{2\text{NA}}$$

where NA = $n \sin(\theta)$ is the numerical aperture of the imaging system and $n$ is the refractive index of the medium and $\theta$ is the half-angle of the cone of light that can be accepted by the lens [2].

1.2 Convolution

For incoherent sources, the image that one will see formed by the imaging system is given mathematically by a convolution between the object and the PSF. In one dimension, the convolution
Figure 1.1: Rayleigh Criterion [3].

is given by

\[ C(x) = \int_{-\infty}^{\infty} \text{object}(t) \cdot \text{PSF}(x-t)dt. \] (1.3)

As an example, which will be shown to be quite useful in the next section, consider a PSF that is a sum of two normalized Gaussian functions having different heights and widths yet centered at the same point. Such a PSF would be

\[ \text{PSF} = \frac{1}{\sigma_1 \sqrt{\pi}} \exp\left(\frac{x^2}{\sigma_1^2}\right) + \frac{1}{\sigma_2 \sqrt{\pi}} \exp\left(\frac{x^2}{\sigma_2^2}\right). \] (1.4)

Next, convoluting the PSF with an object that is a three-bar square wave of width \( a \) yields,

\[ \text{object}(x) = \begin{cases} 1 & \text{if } -5a/2 \leq x \leq -3a/2 \\ -a/2 \leq x \leq a/2 & 3a/2 \leq x \leq 5a/2 \\ 0 & \text{otherwise}. \end{cases} \] (1.5)

The result of this convolution is a sum of error functions, and the image is therefore

\[ \text{Image}(x) = \frac{1}{2} \sum_{i=1,2} \sum_{j=1/2,3/2,5/2} \left[ \text{erf}\left(\frac{x + ja}{\sigma_i}\right) + \text{erf}\left(\frac{x - ja}{\sigma_i}\right) \right], \] (1.6)

which will later prove to be useful for fitting CTF data.

1.3 Transfer Functions

The PSF is often not conveniently determined directly from an imaging system. It is typically easier to make a measurement of the quality of an image in Fourier space, yielding what is called the Optical Transfer Function (OTF), which is a measure of how well the contrast of object is transferred to the image. The OTF consists of two parts; a modulus, called the Modulation Transfer Function (MTF) and a phase, called the Phase Transfer Function (PTF). For an system using a CCD camera to capture the image, the phase information is lost, and one measures
directly the MTF. The MTF measures the response of an imaging system to objects that are decomposed into sine waves. Understanding the MTF then makes it possible to find the PSF by means of the Fourier transform:

\[
\text{OTF}(k) = \text{MTF}(k) \cdot \text{PTF}(k) \quad (1.7a)
\]

\[
\text{MTF}(k) = \int \text{PSF}(x) e^{-ikx} dx \quad (1.7b)
\]

The MTF for a diffraction limited system is given in terms of the spatial frequency, \( f \), and the cut-off spatial frequency, \( f_0 \) by [4]

\[
\text{MTF} = \frac{2}{\pi} \left[ \cos^{-1} \left( \frac{f}{f_0} \right) - \frac{f}{f_0} \sqrt{1 - \left( \frac{f}{f_0} \right)^2} \right]. \quad (1.8)
\]

Another useful transfer function which will be used later is the Contrast Transfer Function (CTF). The CTF measures response of an imaging system to objects that are decomposed into square waves. The CTF and MTF are related by [2, 5]

\[
\text{MTF}(f) = \frac{\pi}{4} \left[ \text{CTF}(f) + \frac{1}{3} \text{CTF}(3f) - \frac{1}{5} \text{CTF}(5f) + \cdots \right]. \quad (1.9)
\]
Chapter 2

Imaging Experiment

This chapter is intended to give a detailed overview of the experimental components and setup used to estimate the resolution of the following imaging system. The illumination technique that is used is called Kohler Illumination which was first introduced in 1893 by August Kohler and is used today as the recommended technique for modern laboratory microscopes [7]. We explore two different ways of evaluating the performance of an imaging system. The first is to simply look at a series of bar patterns with different spatial frequencies that in turn gives the CTF of the system. Secondly, we use the slanted edge method, which requires only a single image of a slanted edge from which one can determine the MTF. Three light sources are used for this experiment: a filament lamp with wavelength 405nm, and two LED’s with wavelengths of 420nm and 700nm. The lenses used in the Kohler illumination setup are plano-convex, which are designed for either focussing or collimating light, depending on which side the light is incident upon.

2.1 Kohler Illumination

The imaging system used in this experiment is based on the principle of Kohler illumination. This technique is used to provide even illumination over the sample of interest, eliminating what is known as the “filament image” [7]. The filament image is an image of the filament of the light source, which results in uneven illumination if simply shone through the objective or a single lens. The general setup of the experiment is shown in Fig. 2.1(a). The light from the source is first collected by the collector lens ($f = 50\, mm$) which focusses down at the condenser aperture diaphragm (see Fig. 2.1), forming an image of the filament. Opening and closing the condenser aperture allows one to control the angles of light that propagate to the sample [7], which results in a change in contrast. When the aperture is opened, there is better contrast due to the higher angles of light rays striking the sample.

From this point, the light is collimated by the second lens ($f = 100\, mm$), after which is placed the field diaphragm at a distance equal to the focal length. Finally, the third lens ($f = 100\, mm$) focusses the light down to the back focal plane of the Zeiss LD-Plan NeoFluar 63x/0.75 objective, acting as the substage condenser in a microscope. In a microscope, the substage condenser serves to gather light from the light source and focuses it onto the specimen [8]. For Kohler illumination,
the substage condenser collimates the light rather than focussing it. Since the light is collimated when it strikes the sample, there is no filament image that appears in the object plane.

The light then reflects off of the sample and is captured and collimated by the objective. A 164.5mm focal length tube lens, placed $100 \pm 10$mm behind the objective, focusses the reflected light onto the Pixelfly QE camera (from PCO, pixel size $6.45\mu m \times 6.45\mu m$).

![Image](image.jpg)

(a) Our Experimental Setup 
(b) Schematic of Kohler Illumination [9]

Figure 2.1: Kohler Illumination

### 2.2 CTF Method

The CTF of the imaging system is determined by using an MRS-5 Geller standard; a scanning electron microscope standard with square wave pitch patterns as small as 80nm. The patterns are etched into a 100nm tungsten film, behind which is a thin layer of SiO$_2$ [10].

To determine the CTF, an image is taken for several different frequency patterns (the largest being $3\mu m$, and the smallest being 500nm). The average of each row (or column, depending which pitch was chosen) of the three bar pattern was used to obtain the blue curves shown in Figs. 2.2(a) and 2.2(b). This data would subsequently be fit to the theoretical result of the convolution of a Gaussian-shaped PSF with a square wave given in Eq. 1.6. The reasoning behind using a double-Gaussian PSF was because it better fit the data for higher spatial frequency patterns, where features such as those in Fig. 2.2(b) show a lower value for the middle minimum than the two outer minima. The fact that two Gaussians were needed for the fit also suggests the presence of spherical aberration, resulting in a large broadening of the base of the PSF. It was later attempted to back-out a resolution from the double-Gaussian fit, although this proved
difficult since information was in both Gaussian functions, therefore the Rayleigh criterion did not produce reliable results.

From the fit, the minimum and maximum values are taken from within the pattern, that is, the “tails” on either end of the pattern are ignored; they are simply there to achieve a good fit. The CTF for each pattern yields a single point, calculated by $I_{\text{max}} - I_{\text{min}} / I_{\text{max}} + I_{\text{min}}$. Each point is then normalized to the 3µm pattern along the short edge, shown in Fig. 2.2(c). This proved to be the most effective way for normalizing the data.

![Graphs](a) 3µm pitch.  (b) 600nm pitch.  (c) Normalizing of the CTF Data.

Figure 2.2: Example Fit to the 3µm and 600nm Pitches.

### 2.3 Slanted-Edge Method

The slanted edge method for determining the resolution properties of an optical imaging system is a widely accepted method [11]. It has proven to be very quick at determining the edge spread function (ESF), the line spread function (LSF) and the MTF. This method requires a
single image of a slanted edge, as opposed to measuring the response of the system to several sinusoidal patterns, as was done to obtain the CTF. The slant, as opposed to imaging a straight edge, allows for a greater sampling along the edge. The edge used for this experiment was a piece of silicon with a thickness of 0.53mm.

A sample edge is given in Fig. 2.3(a), where red indicates high intensity (the silicon) and blue indicates low intensity (no silicon). The first step in the analysis consists of determining the angle of the edge by first performing a gradient operation on the image and then passing the gradient through a radon transform. The radon transform determines the angle and subsequently each row of the 2D matrix is re-projected along the angle of the edge to acquire the true ESF. From this point, the edge is mapped into a vector, sampled as \( p[(i - 1) - (j - 1) \sin(\theta)] \), where \( p \) is the pixel dimension, \( i \) is the row number, \( j \) is the column number and the top-left corner is taken as the (0,0) point. The points are then sorted in ascending order according to the distance at which each pixel falls from the edge (Fig. 2.3(b)). Next, the points are binned. Binning the points allows for a balance between reducing noise while still allowing a good sampling of the spatial frequency. In this work, images are taken to be 100×100 pixels in size with each bin containing 50 points. Fig. 2.3(c) shows the binned ESF. The binned ESF is then numerically differentiated, yielding the LSF (Fig. 2.3(d)). To further reduce high frequency noise at the edges of the LSF, a Hanning window is applied twice. At this point, a fit is made to the data, assuming a Gaussian function for the LSF with a baseline at zero. From the fit, the Rayleigh criterion is invoked to obtain an estimate of the resolution. Finally, the MTF is calculated by the Fast Fourier Transform (FFT) of the LSF, then normalized, for which the resolution is estimated once again by the value of the MTF at 7%.

### 2.4 Software

The software used for focussing the images was Camware, the standard software that came with the PixelFly camera. The acquisition of the image were taken using pycamera.py (see Tout’s report, Ref. [12]). Two MATLAB scripts were used for the analysis of the data, one for the CTF method and the other for the slanted edge method. CTF.m and MTF.m were the scripts used to acquire the CTF’s and MTF’s, respectively.
Figure 2.3: Example of the Slanted Edge Method. $\lambda = 420\text{nm}$. 

(a) Image of the Edge

(b) Edge Spread Function

(c) Binned and Sorted Edge Spread Function

(d) Line Spread Function
Chapter 3

Results and Discussion

In this section, the CTF and the MTF results are presented and commented upon. Questions related to the quality of the transfer function, possible coherence of the light sources, the apparent appearance of coherence due to source sizes and optical aberrations are considered. Imaging is done both with and without cover slips (windows) and the effective numerical aperture of the imaging system is changed to see its effects on performance. We first look at the CTF data, followed by the MTF produced by the slanted edge method and finally the possible types of aberrations that may be present.

3.1 CTF Method

The first set of results presented here are the CTF data. The blue points are results for the 405nm light, and the red points for the 700nm light. We notice immediate a striking difference in the qualitative shape of the CTF. The blue curve drops quickly at small spatial frequencies, and seems to level off after about $f \approx 1.1 \mu m^{-1}$. This behavior is not typical of a diffraction limited system, where the CTF falls more gradually to zero. The red curve on the other hand does exhibit the qualitative behavior that one would expect of a CTF, except for the points at $f = 0.33 \mu m^{-1}$ and $f = 0.5 \mu m^{-1}$ being nearly identical.

The CTF results seem to contradict what would be expected from blue and red light. That is, one would expect the CTF for blue light to be higher than that of red light, since it has a smaller wavelength and should better resolve smaller objects. A possible solution to this question is the quality of the objective being used. The Zeiss LD-Plan Neofluor 63x/0.75 has four-star color correction out of five, which means that it is better corrected for aberrations if the green and red wavelengths than for blue. This point will be emphasized and elaborated upon in the following sections.

The resolution of the CTF can be estimated by its value at about 14%. For the blue light, we can estimate a resolution of about 0.72μm. For the red light, the CTF at 14% is about 0.69μm.
3.2 Slanted Edge Method

The following figures present the results of MTF data acquired by the slanted edge method. In these figures, the blue points represent the MTF data, the dashed green line is the MTF from the fit of the LSF and the red line is the diffraction limited MTF. The diffraction limited MTF was drawn with $\text{MTF}(\frac{\lambda}{2NA})^{-1} = 0$, which is equivalent to $\text{MTF}(1/\text{resolution}) = 0.07$. The corresponding Rayleigh criterion for the MTF is that the resolution is given at about 7%.

Fig. 3.2 shows the MTF for a blue filament light (large LED) with wavelength $\lambda = 405\text{nm}$. Fig. 3.2(a) was taken with the condenser aperture fully opened and Fig. 3.2(b) was taken with the condenser aperture closed down to about 4mm in diameter. One immediately notices that neither of the MTF’s follow the diffraction limited curve, suggesting the presence of optical aberration. A particular feature to make note of is the modulation at “higher” spatial frequencies, that is, from about $f = 1.2\mu\text{m}^{-1}$ to about $f = 3\mu\text{m}^{-1}$. When the condenser aperture is closed down, the modulation is actually higher than when it is fully opened.

The resolution from the fit of the LSF gives a resolution of about $1.3\mu\text{m}$ with the aperture fully opened and about $0.71\mu\text{m}$ with the aperture closed down. The MTF at 7% yields resolutions of $0.80\mu\text{m}$ with the aperture fully opened and about $0.40\mu\text{m}$ with the aperture closed down. The disagreement between the fit of the LSF and the MTF at 7% is quite large. The fit of the LSF for the filament source not representative of the data, as seen from the MTF. This fitting inaccuracy appears to be a phenomenon of the extended blue source only, as we shall see when using the LED’s as sources.

Next the MTF’s are obtained with two different LED light sources (Fig. 3.3). The blue
Figure 3.2: MTF for the Blue Filament ($\lambda = 405\text{nm}$) with the Field Diaphragm Open and Closed.

LED has a wavelength of about $\lambda = 420\text{nm}$ and the red LED about $\lambda = 700\text{nm}$. The blue MTF for the LED (Fig. 3.3(a)) is better than that of the filament source, i.e. it more closely resembles the diffraction limited curve although still suggesting optical aberration is present. This could be due to one or both of the following reasons: First, it is known that the Zeiss objectives with four-star color correction are better corrector for wavelengths in the green and red regime than that for blue. The blue LED has a longer wavelength than the blue filament source, and therefore could be slightly better corrected for aberrations. Second, the blue LED is a much smaller source, better resembling a point source than does the filament. Either of these two factors could contribute to the better resolution obtained by the blue LED, which is about $0.52\mu\text{m}$ from the fit of the LSF and about $0.50\mu\text{m}$ from the MTF at 7%.

The fit of the LSF for the red LED gives a resolution of about $0.63\mu\text{m}$ and about $0.68\mu\text{m}$ from the MTF at 7%. The hump-like feature that is seen at low spatial frequencies likely due to diffraction near the edge (a dark line would appear at the edge as though it were a ripple due to diffraction). However, the fact even the fit from the LSF is better than diffraction limited for the red LED seems to indicate that there is some other phenomena. The first idea that emerges is the coherence of the light. It is known that for perfectly coherent light, the MTF is simply a step function. Thus, considering that the slope of the MTF is larger than that of the diffraction limited curve for incoherent light, it could be that there is some partial coherence of the red LED light. Additionally, in Ref. [13] a correlation is made between the apparent coherence of the light and the size of the source. The conclusion, as shown in Fig. 3.4 is that for “large” source sizes, the MTF approaches that of the incoherent case, and for “small” source sizes, the MTF approaches that of the coherent case. In the figure, $\Delta s$ is the size of the source and $\Delta x$ is the size of a filter or aperture placed in the Fourier plane. Thus, when $\Delta s > \Delta x$, the apparent transfer function follows the incoherent MTF, whereas as $\Delta s$ gradually becomes smaller and smaller, it approaches the coherent case.

Images with the same two LED’s were then taken through a $200\mu\text{m}$ sapphire window. The
Figure 3.3: MTF for the Blue ($\lambda = 420\text{nm}$) and Red ($\lambda = 700\text{nm}$) LED's.

Figure 3.4: Spatial Coherence Apparent Transfer Function [13]
resolutions for the blue and red from the fit of the LSF are about 0.61\( \mu \)m and 0.65\( \mu \)m, respectively. As for the shape of the MTF, it appears that for both LED’s there is a steeper fall-off of the MTF a lower spatial frequencies when imaging through the window. At higher spatial frequencies, the MTF is practically the same. Looking at the resolution from the MTF at 7\%, the data gives a resolution of about 0.52\( \mu \)m for the blue and about 0.67\( \mu \)m for the red. Thus, the general conclusion is a slightly worse resolution when imaging through the 200\( \mu \)m sapphire window.

Figure 3.5: Blue and Red LED Light through a 200\( \mu \)m Sapphire Window with NA = 0.75.

Since neither of the curves above follow the diffraction limited MTF, the idea was then to reduce the numerical aperture by placing an iris in the Fourier plane, between the objective and the tube lens, and to close it down incrementally. Knowing the magnification of the objective and the focal length of the tube lens, it is possible to calculate the effective focal length of the objective:

\[ M = \frac{f_{\text{tube lens}}}{f_{\text{objective}}} \]

\[ f_{\text{objective}} = f_{\text{tube lens}} \cdot \frac{M}{2f_{\text{objective}}} = \frac{164.5\text{mm}}{63} = 2.61\text{mm} \]  

This allows us to calculate the numerical aperture using

\[ \text{NA} = \frac{d}{2f_{\text{objective}}} \]

where reducing the aperture diameter to 1mm yields an effective NA = 0.19. The results in Fig. 3.6 are for an aperture diameter of 1mm.

The MTF for the blue LED appears to some extent follow the diffraction limited curve. It is clear however that the data presents a sharper fall-off than the diffraction limited curve, which suggests once again that there may be some partial coherence of the light source. The blue light gives a resolution of about 1.4\( \mu \)m from the fit of the LSF and about 1.5\( \mu \)m from
the MTF at 7%, where the diffraction limited resolution is about 1.35µm for the incoherent diffraction limited MTF. The diffraction limited resolution for perfectly coherent light would be \( \frac{\lambda}{NA} = \frac{420\text{nm}}{0.19} = 2.2\mu m \). Our experimental result falling between these two diffraction limited resolutions is consistent with partial coherence, which is also suggested from the qualitative shape of the data.

A series of MTF’s were then taken for different numerical apertures with no window (Fig. 3.6(b)). Once again, this was done by changing the diameter of the iris which was located between the objective and the tube lens. The diameter was changed from as small as 1mm up to 4mm in increments of approximately 0.25mm. This corresponds to a numerical aperture ranging from 0.19 to 0.75. As the iris is closed down in the Fourier plane, the higher angles of light rays are effectively eliminated, and the modulation (or contrast) decreases more rapidly. Consequently, the resolution is worse for smaller numerical apertures, yet does appear to approach that of a diffraction limited system.

Note here that, for consistency, the same region was taken for each of the images. Since the edge is not perfectly perpendicular to the objective, there are regions with more noise than others. In the particular region that this experiment was conducted, there was a considerable amount of noise and therefore does not represent the full capability of the imaging system. The main feature to note here is the hump-like feature in Fig. 3.6(c) that emerges at a numerical aperture of 0.43. This is an unexpected feature for a diffraction limited system, and therefore could be suggesting the presence of spherical aberration. When the numerical aperture is very small, spherical aberration could be “blocked” out by the iris by effectively chopping out the higher angles of light near the edges of the lenses in the objective. Once those angles are allowed to reach the camera, and if spherical aberration is present, this would result in deterioration of the image quality.

<table>
<thead>
<tr>
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<th>( \lambda = 405\text{nm} )</th>
<th>( \lambda = 420\text{nm} )</th>
<th>( \lambda = 700\text{nm} )</th>
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<tr>
<td>Diff. Lim. Res. (NA = 0.75)</td>
<td>0.33( \mu m )</td>
<td>0.34( \mu m )</td>
<td>0.57( \mu m )</td>
</tr>
<tr>
<td>Diff. Lim. Res. (NA = 0.19)</td>
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<td>1.35( \mu m )</td>
<td>—</td>
</tr>
<tr>
<td>No Window (NA = 0.75)</td>
<td>1.3( \mu m ) (open), 0.71( \mu m ) (closed)</td>
<td>0.52( \mu m )</td>
<td>0.63( \mu m )</td>
</tr>
<tr>
<td>No Window (NA = 0.19)</td>
<td>—</td>
<td>1.4( \mu m )</td>
<td>—</td>
</tr>
<tr>
<td>200( \mu m ) Window (NA = 0.75)</td>
<td>—</td>
<td>0.61( \mu m )</td>
<td>0.65( \mu m )</td>
</tr>
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Table 3.1: Summary of Resolutions from the Fit of the LSF.

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<tr>
<th></th>
<th>( \lambda = 405\text{nm} )</th>
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<td>—</td>
</tr>
<tr>
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<td>—</td>
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<tr>
<td>200( \mu m ) Window (NA = 0.75)</td>
<td>—</td>
<td>0.52( \mu m )</td>
<td>0.67( \mu m )</td>
</tr>
</tbody>
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Table 3.2: Summary of Resolutions from MTF Data at 7%.
Figure 3.6: MTF Data at Various Numerical Apertures, $\lambda = 420$nm.
3.3 Discussion

3.3.1 CTF Method

The CTF method used is a good way to determine how small of an object is visible, since one is directly imaging that size. It does not, however, allow one to gain accurate information about the resolution of the imaging system, since this would require many intermediate sized patterns that are not available on a single standard. This is, however, the most direct way for testing an optical imaging system.

3.3.2 Slanted Edge Method

The Rayleigh criterion for the resolution determination is an arbitrary one. The resolution of an imaging system strongly depends on the performance of each of its elements in addition to the noise levels, and therefore differs from system to system. Others (such as Ref. [11]) choose their resolution at the 50% value of the MTF, yielding a more conservative value.

It is also worth mentioning that the slanted edge method is only one way to determine the MTF and that other methods, such as the slit method [11] and the knife edge method [14] can be used. The slit method consists of two edge placed very close together, creating a very thin slit for light to shine through. The knife edge method is quite similar to the slanted edge method, however a scan is done along the line perpendicular in order to the edge to acquire the ESF. In order to obtain a very good estimate of the resolution, it would be worth implementing more than one of these methods to check for consistency. Ref. [11] makes a detailed comparison between the slanted edge method and the slit method. They conclude that the slanted edge method measures a better response for low frequency modulation because there is a large number of quanta that contribute to the tails of the LSF. On the other hand, the slit method measures a better response for higher frequency modulation because of “noise amplification” that occurs when numerically differentiating the ESF to obtain the LSF. We have, however, substantially reduced high frequency noise in the MTF by applying the hanning window to the LSF in addition to binning. It can be concluded however that the slanted edge method provides a good estimate of the MTF of an imaging system.

Improvements in this technique include possibly passing the gradient of the edge through the radon transform a second time to further increase the accuracy of angle determination. In Ref. [11], the edge is passed twice through a Hough transform (analogous to the radon transform) and an iterative MTF maximization algorithm to further increase the accuracy of the angle.

Additionally, the LSF was fit to a Gaussian function and the resolution was computed using the Rayleigh criterion for the Gaussian fit. Other functions could also be fit to the data, especially for the filament source where the fit did not represent the MTF. It was also attempted to fit the ESF to an error function and to subsequently numerically differentiate the error function fit, although it proved to be more difficult to obtain a good fit for a finite set of ESF data than for the LSF.

3.3.3 Aberrations

The most likely type of aberration present in our imaging system would be spherical aberration, where the light rays from the extremities of a lens are focussed in a different plane than those
rays coming from the center of the lens. This is because we are imaging on-axis, and therefore would not expect other off-axis primary aberrations such as coma and astigmatism to be present. Spherical aberration would also be consistent with the hump-like feature noticed in Fig. 3.6(c) when the numerical aperture is effectively changed. It could also explain the need for the double Gaussian fit for the CTF data, for which the second Gaussian broadened the base of the PSF.
Chapter 4

Conclusions

To conclude, two different methods at determining the quality of an optical imaging system were used, namely the CTF method and the slanted edge method. The CTF method proved to be a good way for determining the response of the system to individual spatial frequency patterns. The slanted edge method proved to be very quick at determining the MTF, requiring only a single image of a slanted edge. The resolutions obtained from a Gaussian fit of the LSF and the MTF at 7% agree with one another to within less than 15% for the LED sources. Additionally, it appears that there is strong evidence for spherical aberration, especially in the blue wavelength regime. This evidence includes the qualitative shape of the CTF and MTF in addition to the hump-like feature that emerges at a numerical aperture of 0.43. Finally, the apparent coherence of the red LED light could possibly be explained by the small size of the source, which is also consistent with the change in MTF for the filament source as the condenser aperture is closed down.
Bibliography


