Design of a magnetic transfer system

Josefine Metzkes

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University of Toronto

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1 Introduction

In this report I am going to present the design of a magnetic transfer system for an ultra cold atom experiment. I have been working on this project in the context of the Research Course PHY6051 with Professor Joseph H. Thywissen at the University of Toronto during the spring term 2008.

My report will start with an introduction of the experimental set-up the transfer system is built for in order to motivate the need for a transfer system. Then I will move on to discuss some foundations of magnetic trapping and magnetic transfer that will be referred to later. In the main part of the paper I will present the results of coil and support system design as well as the final current schemes. Besides, I will try to motivate the results yielded and outline the methods applied.

2 Experiment and Motivation

Objective of the experimental set-up the magnetic transfer system will be embedded in is a quantum simulation of the repulsive Fermi Hubbard model using ultra cold fermionic potassium in a two-dimensional optical lattice. The repulsive Fermi Hubbard model is a possible candidate to explain high-temperature superconductivity. An essential part of this experiment will be in-situ single-site microscopy at the lattice in order to measure spin and occupation statistics especially for disordered filling states. Being non-periodic, they are not accessible to observation by diffraction methods.

An optical lattice experiment with ultra cold atoms alone and in-situ microscopy in special call for specific conditions: Firstly, the experimental site has to allow for very good optical access to bring the lattice beams into the chamber and to leave enough space for observation device like the microscope. Secondly, the vacuum has to be high at the lattice site in order to yield long trapping lifetimes for the atoms.

Those demands however are hardly compatible with the conditions under which the cloud of cold atoms is produce in a magneto-optical trap (MOT). In the MOT, about $10^8$ potassium atoms are captured from a vapor, trapped and cooled to temperatures of some hundred $\mu$K. The cloud is then loaded into a quadrupole trap. The total pressure in the loading chamber is about $10^{-9}$ mbar leading to insufficient trapping lifetimes for longer
experiments. Furthermore, the six MOT beams coming into the vacuum chamber obstruct the optical access.

One possible solution to overcome these problems is to spatially separate the preparatory stage of magneto-optical trapping and cooling on the one hand and the stage of evaporative cooling and actual optical lattice experiment on the other hand. This then allows for differential pumping between the two chambers giving UHV conditions at the lattice site.

Spatial separation of course calls for a possibility to transfer cold atoms between the two vacuum chambers, the loading and the lattice chamber, possibly happening without heating of the cloud or atomic loss from it.

One approach has been presented by Hänsch and co-workers from Munich in 2000 [1]. They magnetically transferred a cloud of $10^9$ rubidium atoms over a distance of 33 cm by moving a quadrupole trap in space. The rubidium atoms having an initial temperature of $125 \mu K$ where heated by less then $20 \mu K$ during the 4 s of transfer between two chamber with a pressure difference of $10^{-2}$ mbar. The clear advantage of this transfer method is the fact that the atomic cloud stays confined in all three spatial dimensions avoiding an expansion during transfer.

The transfer system I have been designing generally follows this approach. However, whereas in Hänsch’s scheme the trapping center is displaced perpendicular to the axis of symmetry of the field producing quadrupole coils, the system described here also uses a scheme where the trapping center is displaced along the axis of symmetry of the coils. In this configuration, neither the loading chamber nor any part of the transfer system are in the plane of the optical lattice allowing for the necessary optical access.

3 Magnetic trapping of neutral atoms

3.1 Magnetic trapping

Magnetic trapping of neutral atoms originates from the interaction between a magnetic dipole moment $\mu$ and an inhomogeneous static magnetic field $B(r)$. The interaction energy $U$ and the force derivable from this potential are given by

$$ U = -\mu \cdot B \quad \text{and} \quad F = -\nabla U = \nabla (\mu \cdot B) . \quad (1) $$
Depending on the sign of $\mu$, a magnetic dipole either feels a force that drives it towards regions of low magnetic field for $\mu < 0$ or to regions of high magnetic field for $\mu > 0$. The first species, low field seekers, can be trapped in a field minimum. High field seekers however are not trappable since Maxwell’s equations forbid the existence of field maxima in regions free of charges and currents [2].

For an atom in a state $|n L S I J F\rangle$ with principal quantum number $n$, quantum numbers $L, S$ and $I$ for the orbital, electronic spin and nuclear spin angular momentum respectively and total angular momenta $\hat{J} = \hat{L} + \hat{S}$ and $\hat{F} = \hat{J} + \hat{I}$, the magnetic moment can be expressed as

$$\mu = g_F m_F \mu_B .$$

(2)
Here $\mu_B$ is the Bohr magneton, $m_F$ the projection of the total angular momentum operator $\vec{F}$ on the magnetic field direction and $g_F$ the Landé $g$–factor given by

$$
g_F = g_J \cdot \frac{F(F + 1) + J(J + 1) - I(I + 1)}{2F(F + 1)} \quad \text{(3)}$$
$$
g_J = 1 + \frac{J(J + 1) + S(S + 1) - L(L + 1)}{2J(J + 1)} \quad \text{(4)}$$

Equating expression (1) for the potential energy $U \approx \mu_B \cdot B$ in the trap with the kinetic energy $k_B T$ of an atom in a cloud at temperature $T$ allows to give an estimate for the trapping depth achievable in magnetic traps. Assuming a reasonable field strength of $B = 100 \text{G}$ yields a the trapping depth of about $2 \text{mK}$. This explains why magnetic trapping of neutral atoms – proposed in 1960 already – could only be realized in 1985 after various laser cooling techniques had been developed providing atoms cold enough [3].

In the experiment described in this report, potassium $^{40}\text{K}$ in the $^2\text{S}_{1/2}$ level is pumped into the $|F = 9/2, m_F = 9/2\rangle$ hyperfine state for magnetic trapping. With a nuclear spin $I = 4$, the Landé $g$–factor can be calculated to $g = 2/9$ for this state so that $\mu = \mu_B$ and the necessary trapping depth for a cloud at temperature $T \approx 100 \mu\text{K}$ after the MOT stage is around $B = 15 \text{G}$. 

### 3.2 The quadrupole trap

One way to classify magneto static traps is according to whether the field magnitude at the trapping center is zero or not. Equation (2) shows, why this classification is useful: At zero field, the hyperfine states are degenerate with respect to $m_F$ so that transitions to non–trapping $m_F$ states become possible causing atom loss in the trap. Those Majorana transitions are avoided for traps with $B \neq 0$ at the trapping center.

A good review of magneto static trapping fields of both types can be found in [4]. Only the quadrupole trap will be discussed here as it will be seen to be the ”unit cell” of the magnetic transfer system. Despite belonging into the first class of traps where the field magnitude vanishes at the trapping center, it will be shown later that Majorana transitions can nonetheless be neglected in our case.

In principle, a quadrupole trap is created from two current loops of radius $R$ placed at a distance $d = 2A$ along a common axis of symmetry. The currents $I$ are equal but counter
propagating. The magnetic field of a single current loop positioned at \( \rho = 0 \) and \( z = A \) is given by

\[
B_\rho(\rho, z) = \frac{\mu_0 I}{2\pi \rho} \cdot \frac{z - A}{\sqrt{(R + \rho)^2 + (z - A)^2}} \cdot \left[-K(k^2) + \frac{R^2 + \rho^2 + (z - A)^2}{(R - \rho)^2 + (z - A)^2} \cdot E(k^2)\right]
\]

(5)

\[
k^2 = \frac{4R\rho}{(R + \rho)^2 + (z - A)^2}
\]

(6)

\[
B_z(\rho, z) = \frac{\mu_0 I}{2\pi} \cdot \frac{1}{\sqrt{(R + \rho)^2 + (z - A)^2}} \cdot \left[K(k^2) + \frac{R^2 - \rho^2 - (z - A)^2}{(R - \rho)^2 + (z - A)^2} \cdot E(k^2)\right]
\]

(7)

in cylindrical coordinates where \( E(k^2) \) and \( K(k^2) \) are the complete elliptic integrals and \( \mu_0 \) is the permeability of free space.

Obviously, the magnetic field of a single current loop is axially symmetric with respect to the z axis and so is the field in a quadrupole trap. During horizontal magnetic transfer however, this field symmetry is broken so that it is useful to also express \( \mathbf{B} \) in Cartesian coordinates.

\[
B_x(x, y, z) = B_\rho(\rho \rightarrow \sqrt{x^2 + y^2}, z) \cdot \frac{x}{\sqrt{x^2 + y^2}}
\]

(8)

\[
B_y(x, y, z) = B_\rho(\rho \rightarrow \sqrt{x^2 + y^2}, z) \cdot \frac{y}{\sqrt{x^2 + y^2}}
\]

(9)

\[
B_z(x, y, z) = B_z(\rho \rightarrow \sqrt{x^2 + y^2}, z)
\]

(10)

Rather than giving the exact result for the field magnitude in a quadrupole trap which is readily calculated from the equations for the single current loop, more insight into the field geometry is gained from a Taylor expansion of the field about \( \rho = 0 \) and \( z = 0 \). If the origin of the coordinate system is placed into the center of symmetry of the trap, then \( B_\rho = G_\rho \cdot \rho \) and \( B_z = G_z \cdot z \). The field gradients \( G_\rho \) and \( G_z \) are related through Maxwell’s equation \( \nabla \cdot \mathbf{B} = 0 \) that yields \( G_\rho = -1/2 \cdot G_z \equiv G \). The field magnitude can then be expressed as

\[
B = G \cdot \sqrt{\rho^2 + 4z^2}
\]

(11)

The magnetic field vanishes at the origin and increases linearly in all directions with a spatially varying gradient so that atoms are confined in all three spatial dimensions.

Choosing a specific field gradient for a quadrupole trap influences the size of the atomic cloud as well the cloud dynamics as will be shown in the next sections. Before, one important remark should be made: real quadrupole traps do rarely consist of single current
loops but are made up of magnetic coils. Their field is simply given by a superposition of single current loop fields.

### 3.3 Atomic cloud size in a quadrupole trap

The expression *atomic cloud* that I have been using so far without giving a motivation becomes understandable when the atomic density \( n(\mathbf{r}) \) in a quadrupole trap is studied.

\[
n(\mathbf{r}) = n_0 \cdot e^{-\beta \mathbf{F} \cdot \mathbf{r}} \quad (12)
\]

Here \( \beta = (k_B T)^{-1} \) and \( \mathbf{F} = \mu_B (G_x, G_y, G_z) \) is the force on a dipole \( \mu = \mu_B \) in a magnetic field with gradients of magnitude \( G_i \). Using the normalization condition \( \int n(\mathbf{r}) d^3 r = N \) with the total number of atoms \( N \), the peak density \( n_0 \) is calculated to

\[
n_0 = \frac{N}{8} (\beta \mu_B)^3 (G_x G_y G_z) \quad . (13)
\]

A possible measure for the expansion of the cloud in a direction \( x_i \) is the root mean square size \( x_{rms}^i = \sqrt{\langle x_i^2 \rangle} \) given by

\[
x_{rms} = \frac{\sqrt{2}}{\beta \mu_B G_x} \quad , \quad y_{rms} = \frac{\sqrt{2}}{\beta \mu_B G_y} \quad , \quad z_{rms} = \frac{\sqrt{2}}{\beta \mu_B G_z} \quad . (14)
\]

From the discussion of the gradients in a quadrupole trap it follows that the atomic cloud has the shape of an ellipsoid with equal axis lengths in \( x \) and \( y \) direction and half the axis length in \( z \) direction. With a field gradient \( G_z = 100 \text{ G cm}^{-1} \) as it is given in our quadrupole trap and assuming a temperature \( T = 100 \mu \text{K} \), \( x_{rms} = y_{rms} = 0.42 \text{ mm} \) and \( z_{rms} = 0.21 \text{ mm} \). A cloud of this size contains 43% of the atoms whereas 99% of the atoms are enclosed if the cloud is extended to four times the root mean square radii.

In order to truncate a minimum from the atomic distribution and to allow for transfer even in the case of a slight misalignment of tube and transfer coils the diameter of the horizontal transfer tube should be large. The chosen tube diameter which is in variance with vacuum considerations is \( ID = 0.5 \text{ in.} \)

Finally the effects of a modulated gradient will be studied. The equations (14) show that an increasing gradient decreases the cloud size. Consequently, the atomic density \( n \) increases if a fixed volume is considered. At the same time, the temperature of the cloud will increase as the phase space density \( \rho_{ps} \) has to be conserved.

\[
\rho_{ps} = n \cdot \lambda_{dB}^3 = n \cdot \left( \frac{h}{\sqrt{2\pi m k_B T}} \right)^3 \quad (15)
\]
$\lambda_{dB}$ is here the de–Broglie wavelength. The relation (15) shows that a modulation of the aspect ratio automatically leads to a modulation of the cloud temperature.

### 3.4 Atomic motion in a quadrupole trap

The study of atomic dynamics in a magnetic quadrupole trap will turn out to be essential for an understanding of limiting parameters in magnetic transfer, e.g. the maximum transfer velocity. I will start with a discussion of classical orbits and justify in the following why quantum effects are negligible in the regime we are working in.

Insight into atomic motion in a quadrupole trap can already be gained from a very simple model where the motion is considered to be confined to the $z=0$ plane. Every stable orbit within this plane has to fulfill the condition

$$\mu_B G_\rho = \frac{mv^2}{\rho}, \quad (16)$$

the classical equation of motion for a particle of mass $m$, here the atomic mass, moving in an orbit of radius $\rho$. The angular frequency of an orbit is readily calculated from the velocity $v$ of the atom and the orbital circumference:

$$\omega_T = \sqrt{\frac{\mu_B G_\rho}{m\rho}}. \quad (17)$$

The velocity of atoms can be deducted from the temperature of the atomic cloud using the equipartition theorem $E_{kin} = \frac{mv^2}{2} = \frac{3k_B T}{2}$. For $^{40}K$ atoms at $T = 100 \mu K$ in a field gradient $G_\rho = 50 \text{Gcm}^{-1}$, the velocity is approximately $v \approx 0.25 \text{ms}^{-1}$ yielding an average orbital radius of $\rho \approx 0.9 \text{mm}$ from equation (16). The corresponding orbital frequency is $\omega_T/2\pi \approx 40 \text{Hz}$.

A second time scale for the trap dynamics is given by the Larmor frequency $\omega_L = \mu B/\hbar$, the precession rate of the magnetic moment in the field. Determining the time a magnetic moment $\mu$ needs to realign with a spatially varying field direction, we can use $\omega_L$ to give a criterion for adiabacity:

$$\omega_L \gg \frac{1}{B} \left| \frac{dB}{dt} \right| = \omega_T. \quad (18)$$

The second equality holds for a constant field gradient. A violation of the adiabacity condition (18) leads to losses from the trap when atoms make transitions to non–trapping states in a quickly changing field. Using the orbital radius and $G_\rho = 50 \text{Gcm}^{-1}$ in equation
(11) yields $\omega_L/2\pi \approx 6\text{ MHz}$ showing that the assumption of adiabacity is fully valid in our case. Non–adiabatic behavior sets on when $\rho \approx (\hbar^2/m\mu_B G_\rho)^{1/3} \approx 1\mu\text{m}$ corresponding to a temperature $T \approx 100\text{ nK}$. The same length scale marks the transition from classical to quantum dynamics [5] as the atom is confined to regions comparable to its de–Broglie wavelength. Working at temperatures that are three orders of magnitude higher justifies the classical treatment applied here.

A final remark refers to the range of validity of the results derived above: Closed orbits as shown only exist for motion confined to either the $\rho = 0$ or the $z = 0$ plane. All other orbits experience a non–central force so that angular momentum is not a conserved quantity anymore. Those consequentially non–closed orbits can be calculated numerically [6].

4 Magnetic transfer of neutral atoms

In this chapter it will be shown how a cloud of neutral atoms cannot only be trapped but also be spatially displaced using quadrupole traps. During the transfer, the cloud will be confined in all three dimensions as in a simple quadrupole trap circumventing an expansion of the cloud. By adjusting the currents in the coils, the transfer as well as the trapping properties can be controlled.

4.1 Horizontal transfer with two coils

The transition from a quadrupole trap to a transfer unit is made by combining two partly overlapping quadrupole traps. By adjusting the relative currents in the two traps, the trapping center can be shifted in space. Having two adjustable currents, two degrees of freedom of the transfer can be controlled, the first obviously being the position of the trapping center in space. The discussion of the cloud size has shown that the vertical gradient is a second important parameter since it influences the cloud size. Studying the transfer between the two traps in more detail reveals however the drawback that the ratio of the gradients in horizontal direction – the aspect ratio – changes leading to a modulation of the cloud size and consequently cloud temperature. The aspect ratio is defined as $A = G_x/G_y$ where $G_x$ and $G_y$ are the field gradients in $x$ and $y$ direction respectively where the latter will be the horizontal transfer direction from now on.
Figure 2: Continuity of horizontal transfer in dependence of coil overlap

When the cloud is trapped in a single pair of coils, the aspect ratio is obviously one due to the axial symmetry of the magnetic field. If the same currents are flowing in both coil pairs, the trapping center is displaced to the overlap region of the coils. In that region the fields generated by the two traps cancel out leading to a trap geometry elongated in the direction of transfer. Further transfer would decrease the aspect ratio back to one and then increase again. This problem can easily be solved for most parts of the horizontal transfer distance by adding a third active pair of coils at every instance in time.

Before discussing the three–coil–scheme it is important to clarify what is meant by overlapping coil. Denoting the distance between the two centers of the coil pairs by \( r_{ov} \), continuous transfer is possible for \( r_{ov} \leq R \) where \( R \) is the coil radius. Continuous transfer here means that the trapping center moves approximately equal distances in space for equal changes in currents over the whole transfer distance. This is impossible for non–overlapping coils meaning \( r \geq 2R \) and the region of intermediate overlap \( R < r < 2R \) the atomic cloud starts to be displaced erratically. Since a shaking of the cloud during transfer might lead to trap losses, coils with an overlap \( r = R \) will be used in future.
4.2 Horizontal transfer with three coils

Adding a third coil at every instant in time allows for control of three parameters which will be the position of the trapping center, the vertical gradient and the aspect ratio. The trapping geometry is then constant along the whole transfer distance but now with an aspect ratio always larger than one corresponding to an elongated trap.

As well as the vertical gradient the aspect ratio can be chosen as it is convenient. Experimenting with different aspect ratio it can be seen that depending on the coil geometry there is one lowest aspect ratio that allows all horizontal transfer coils to be unipolar. This aspect ratio is chosen for the transfer.

4.3 Vertical transfer

Whereas in the horizontal transfer scheme the trapping center is moved perpendicular to the axis of symmetry of the traps, in vertical transfer the trapping center is displaced along the axis of symmetry. Consequently, the field symmetry is not disturbed and the aspect ratio is one during the transfer automatically. Therefore, two coil pairs with two different currents are sufficient to control the trapping center position and the vertical gradient.

In the basic vertical transfer scheme we use four coils that are positioned along a common axis of symmetry. Coil 1 and 3 and 2 and 4 respectively are paired up to form a quadrupole trap. Transfer from the first to the second quadrupole trap is achieved by simply decreasing the currents in the first pair of coils while increasing the current in the second pair of coils. The disadvantage of this conceptually more simple scheme is however that bipolar coils are needed if transfer not just between two quadrupole traps is of interest. Then every coil acts as upper coil first and eventually as lower coil in a quadrupole trap or vice versa equivalent to a change in flow direction of the current. Looking at the current schemes calculated for the vertical section of our transfer system this becomes obvious.
5 Coil design

The whole transfer system consists of 27 single coils with five different geometries: a single push coil, a pair of MOT coils, 9 pairs of horizontal transfer coils, four vertical transfer coils and a single coil pair for evaporative cooling.

5.1 Design of single coils

The choice of a specific coil geometry depends in our case first of all on the vertical gradient the quadrupole trap shall produce. For a single–loop trap the vertical field gradient $G_z$ close to the trapping center and on the axis of symmetry of the quadrupole trap is given by

$$G_z(\rho = 0, z \approx 0) = \frac{3I\mu_0}{2} \cdot \frac{d \cdot R^2}{(\frac{d^2}{4} + R^2)^{5/2}}$$

where $d$ is the separation of the two coils as above. Given a fixed current $I$ and a fixed coil separation $d$ for the trap, the gradient $G_z$ is maximum when holds $R = d/\sqrt{6}$; vice versa, the gradient peaks if $d = R$ is chosen for a fixed current and a fixed radius $R$.

In the case of a multiple–loop coil these results above apply to the mean radius $\bar{R} = (r_i + r_o)/2$ of inner and outer radius.

Furthermore, the coil geometry should be optimized with regard to a minimum power consumption. The power in $W$ dissipated by a single coil is given by

$$P = R \cdot I^2 = \frac{\rho}{A} \cdot I^2 \cdot \sum_{n=1}^{N} 2\pi r_n \approx \rho \cdot \frac{2\pi \bar{R} N}{A} \cdot I^2 ,$$

where $R$ is the resistance of the coil depending on the resistivity $\rho^1$, the area $A$ of the wire and the total wire length. The latter is given by the sum over the circumferences of all current loops with radii $r_n$. Instead the average radius $\bar{R}$ times the number $N$ of loops can be used. Since the power (20) increases quadratically in $I$ but only linearly in the number of loops, it is usually more efficient to add further wrappings instead of choosing higher currents.

In the following, the choice of the horizontal and vertical transfer coils will be explained. MOT, push and gradient coils will not be considered since their geometry might still be subject to change as the MOT and lattice chamber are designed.

\footnote{For copper $\rho = 1.78 \cdot 10^{-8} \Omega \text{m.}$}
**Horizontal transfer coils**

The design of the horizontal transfer coils is constrained by a valve in the horizontal transfer tube that demands a minimum separation of $d = 50.0$ mm between the inner layer of horizontal transfer coils. The optimum mean radius is then given by $R = 20.4$ mm and the desired gradient of $G_z = 100$ G cm$^{-1}$ is efficiently yielded by setting $r_i = 12.4$ mm and $r_o = 30.48$ mm and using two axial layers of wire.

**Vertical transfer coils**

Whereas the horizontal transfer coils will be mounted to the vacuum system in a way that allows to remove them during e.g. baking of the apparatus, the vertical transfer coils will be stacked on a vacuum tube and stay in place permanently. For that reason, heat insulation between the vacuum tube and the coil is an important issue that is considered in the design by making the inner diameter of the coil wide: $r_i = 25.0$ mm. With a separation of $d = 60.0$ mm between the coils in the quadrupole trap the required gradient of $G_z = 100$ G cm$^{-1}$ can be produced by using $r_o = 104.3$ mm and wrapping four axial layers of wire. The coil separation $d$ chosen is the maximum coil separation conformable with the given gradient $G_z$ and currents below $I = 50$ A.

**5.2 Design of the complete transfer system**

The total transfer distance is $l_t = 540$ mm where the transport is horizontally over a distance of $l_h = 360$ mm and vertically over a distance of $l_v = 180$ mm. The horizontal length is determined by the purpose of the transfer system to clear the view ports of the lattice chamber from all parts of the MOT chamber. Furthermore, a minimum tube length is needed in order to achieve a given pressure difference between MOT and lattice chamber in differential pumping. The vertical length has been chosen so that the view ports of the lattice chamber lie above the plane of the horizontal MOT beams.

As it has been shown above, the more overlap neighboring coils have, the more continuous the horizontal transfer will work. For this reason, the overlap of the horizontal transfer coils has been maximized to $r_{ov} = r_o$ where $r_o$ is the outer coil radius. $r_{ov}$ is here the distance between the two coil centers. The overlap is however less where coils of different sizes connect, namely at the transition from the horizontal transfer coils to the MOT or
the vertical transfer coils. In the first case, at the transition from the MOT, a so-called push coil is used to compensate for this. On the one hand, this coil literally pushes the trapping center from the MOT coil in the direction of the first horizontal transfer coil by adding a bias field. On the other hand, the push coil is needed to fully control the trapping geometry – namely trapping center position, vertical gradient and aspect ratio – by contributing a third degree of freedom as discussed above.

The same issue arises at the transition from horizontal to vertical transfer but simulations have shown that using a pull coil at this place is not practical. For this reason, the aspect ratio of the trap will not be controlled during the last 40.5 mm of the horizontal transfer section and increase up to $A = 3$ in comparison to the designated value of $A = 1.665$. This is acceptable at this position because the cloud will decelerate and stop before starting the upwards motion.

An important remark should be made why the argument of slow transfer does not apply to the MOT coils: Due to differential pumping the pressure decreases along the horizontal transfer tube lowering the probability of collisions. Close to the MOT chamber however the pressure is high so that trap loss producing collisions are likely. They can only be avoided by minimizing the time spent in this region meaning fast transfer.
6 Current schemes

Once the coil geometries and positions within the transfer system are fixed, the remaining degrees to freedom in order to control trapping and transfer are the currents $I$ as equation (5) shows. In this section the current schemes calculated for our transfer system will be presented along with an explanation of how those results have been yielded.

6.1 Method of calculation

The general method for the calculation of currents schemes will be explained using the example of horizontal transfer. As before, $y$ is the direction of transfer, $z$ points along the vertical axis of the traps and the $x$ direction is perpendicular to the $y$–$z$ plane. The origin of the coordinate system $\mathbf{r} = (0, 0, 0)$ lies at the center of the MOT.

The magnetic trapping field will be evaluated at positions $\mathbf{r}_{\text{pos}} = (0, y_{\text{pos}}, 0)$ along the transfer route and is given by

$$B(\mathbf{r}_{\text{pos}}) = \sum_{i=1}^{3} B^i(\mathbf{r}_{\text{pos}}) = \sum_{i=1}^{3} \tilde{B}^i(\mathbf{r}_{\text{pos}}) \cdot I_i$$

(21)

because three coils are active at every instance. The three unknown quantities that have to be determined are the currents $I_1(\mathbf{r}_{\text{pos}})$, $I_2(\mathbf{r}_{\text{pos}})$ and $I_3(\mathbf{r}_{\text{pos}})$. They have to fulfil
the following set of equations:

\[
\begin{align*}
\mathbf{B}(\mathbf{r}_{\text{pos}}) &= 0 \quad (22) \\
G_z(\mathbf{r}_{\text{pos}}) &= \left(\frac{\partial B_z}{\partial z}\right)_{\mathbf{r}_{\text{pos}}} = G_0 \quad (23) \\
A(\mathbf{r}_{\text{pos}}) &= \frac{G_z(\mathbf{r}_{\text{pos}})}{G_y(\mathbf{r}_{\text{pos}})} = \frac{\left(\frac{\partial B_z}{\partial x}\right)_{\mathbf{r}_{\text{pos}}}}{\left(\frac{\partial B_y}{\partial y}\right)_{\mathbf{r}_{\text{pos}}}} = A_0. \quad (24)
\end{align*}
\]

The first equation shifts the position of the trapping center to \( \mathbf{r}_{\text{pos}} \), the second equation sets the vertical gradient to the designated value \( G_0 \) and the third equation sets the aspect ratio to the fixed value \( A_0 \). In our case the vertical gradient is \( G_0 = 100 \, \text{G cm}^{-1} \) throughout the transfer and as explained earlier, the minimum aspect ratio that allows for unipolar horizontal transfer coils is \( A_0 = 1.665 \) given the coil geometry chosen.

For the implementation into a computer algebra system like Mathematica the equations (22) can be simplified. The magnetic field will be Taylor expanded around \( \mathbf{r}_{\text{pos}} \) and the
relations $B_x(\mathbf{r}_{pos}) = 0$ and $B_z(\mathbf{r}_{pos}) = 0$ holding for symmetry reasons will be used:

$$B_x(\mathbf{r}_{pos}) \approx B^{(1)}_x(\mathbf{r}_{pos}) \cdot x$$  \hspace{1cm} (25)  

$$B_y(\mathbf{r}_{pos}) \approx B^{(0)}_y(\mathbf{r}_{pos}) + B^{(1)}_y(\mathbf{r}_{pos}) \cdot y$$  \hspace{1cm} (26)  

$$B_z(\mathbf{r}_{pos}) \approx B^{(1)}_z(\mathbf{r}_{pos}) \cdot z$$  \hspace{1cm} (27)  

The Taylor expansion coefficients $B_j^{(n)}(\mathbf{r}_{pos})$ are defined by

$$B_j^{(n)}(\mathbf{r}_{pos}) = \frac{\partial^n B_j}{\partial j^n}$$  \hspace{1cm} (28)  

and with equation (21) they can be expressed as

$$B_x^{(1)}(\mathbf{r}_{pos}) = \sum_{i=1}^{3} \left( \tilde{B}_x^i \right)^{(1)}(\mathbf{r}_{pos}) \cdot I_i$$  \hspace{1cm} (29)  

$$B_y^{(0)}(\mathbf{r}_{pos}) = \sum_{i=1}^{3} \left( \tilde{B}_y^i \right)^{(0)}(\mathbf{r}_{pos}) \cdot I_i$$  \hspace{1cm} (30)  

$$B_y^{(1)}(\mathbf{r}_{pos}) = \sum_{i=1}^{3} \left( \tilde{B}_y^i \right)^{(1)}(\mathbf{r}_{pos}) \cdot I_i$$  \hspace{1cm} (31)  

$$B_z^{(1)}(\mathbf{r}_{pos}) = \sum_{i=1}^{3} \left( \tilde{B}_z^i \right)^{(1)}(\mathbf{r}_{pos}) \cdot I_i$$  \hspace{1cm} (32)  

Figure 6: Vertical current scheme
The set of equations yielding the currents $I_i$ at $\mathbf{r}_{pos}$ then transforms to

\begin{align}
B_y^{(0)}(\mathbf{r}_{pos}) &= 0 \\
B_z^{(1)}(\mathbf{r}_{pos}) &= G_0 \\
\left( \frac{B_x^{(1)}}{B_y^{(1)}} \right)_{\mathbf{r}_{pos}} &= A_0 .
\end{align}

and can easily be solved.

\section*{6.2 Current schemes in space}

The resulting current schemes evaluated at positions $\mathbf{r}_{pos}$ for horizontal as well as vertical transfer are shown below together with a plot of the aspect ratio during the horizontal transfer section. Whereas the increasing aspect ratio during the last 40 mm of transfer is due to the fact that this parameter cannot be controlled with only two coil pairs, the increasing aspect ratio at the beginning of the transfer is controlled. Here the transition from a quadrupole trap with $A_{QP} = 1$ to the elongated transfer quadrupole trap with $A = 1.665$ is made.

Another issue is the transition from horizontal to vertical transfer: At the end of the horizontal transfer section the cloud is trapped between coil 12A and 12B whereas the cloud has to be trapped between coil 12A and 13 for vertical transfer. The current in coil 13 has to be switched to a finite value without changing the trapping position of the cloud which is achieved when the currents $I_{12A}$ and $I_{12B}$ simultaneously fulfill the following conditions:

\begin{align}
I_{12A} &= -19.997 - 0.015 \cdot I_{13} \\
I_{12B} &= 19.997 - 0.219 \cdot I_{13} .
\end{align}

The plots show that all horizontal transfer coils as well as the push and MOT coils are unipolar. In the vertical transfer section consisting of six coils four coils are bipolar and two are unipolar. The fact that a maximum of three coil pairs is active at every instance helps to minimize the number of current controls that are needed: Traps that are not used simultaneously can be connected to the same current control. In table below \emph{trap} denotes that a pair of coils in series is meant whereas \emph{coil} refers to a single coil.

The total current at every instance in time that needs to be know to layout the power supply is shown below.
<table>
<thead>
<tr>
<th>Power control 1</th>
<th>Power control 2</th>
<th>Power control 3</th>
<th>Power control 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trap 1</td>
<td>Trap 2</td>
<td>Trap 3</td>
<td>–</td>
</tr>
<tr>
<td>Trap 4</td>
<td>Trap 5</td>
<td>Trap 6</td>
<td>–</td>
</tr>
<tr>
<td>Trap 7</td>
<td>Trap 8</td>
<td>Trap 9</td>
<td>–</td>
</tr>
<tr>
<td>Trap 10</td>
<td>Trap 11</td>
<td>Coil 12 A</td>
<td>Coil 12 B</td>
</tr>
<tr>
<td>Coil 13</td>
<td>Coil 14</td>
<td>Coil 15</td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Grouping of the traps for current control

![Total current for horizontal transfer with G = 100 G/cm](image)

Figure 7: Total currents during horizontal transfers

### 6.3 Current schemes in time

In order to define the transfer scheme in time, the trap position $r_{trap}$ as a function of time has to be determined. Given that $r_{trap}(t)$ is known, I suggest the following steps that yield the currents $I(t)$:

1. For each coil, the current schemes $I(r)$ is approximated by an analytical function$^2$.

The corresponding results can be found in the folders `Functions for currents_Horizontal` and `Functions for currents_Vertical`. In some cases more than one function

---

$^2$For the push coil, no analytical function could be found so that the values have to be interpolated using linear functions.
had to be used to get a useful approximation. Before working with those analytical current scheme functions I would consider it important to compare the trapping geometry produced by the exact and approximative current values. Important characteristics to look into would be the position of the trapping center, the vertical gradient and the aspect ratio.

2. Replace \( r \) by \( r(t) \) in the functions for the current schemes in order to determine the current of each coil at times \( T \) to program the power supply.

I suppose that it would be useful to write a Mathematica script (or similar) where all functions \( I(r) \) as well as \( r(t) \) are implemented so that the matrix containing the currents for all coils at times \( T \) can easily be extracted even for changing current or time patterns. The function for the time–dependent trap position \( r_{\text{trap}}(t) \) has not been determined yet but some rough characteristics are already known. The cloud will be accelerated out of the MOT, then transfered with a constant velocity before it is decelerated and stops at the transition to the vertical transfer section. Then acceleration towards the science chamber and deceleration towards the final position between the gradient coils follow.

The first step would be to determine the acceleration as a function of time so that the function for \( r_{\text{trap}}(t) \) can be yielded through integration. In order to avoid infinite forces
on the cloud, the acceleration should change linearly in time which is equivalent to a constant jerk \(^3\). For this fact, see for example the drawings on page 8 in reference [11]. A reference value for the maximum acceleration can be found in [8] given by \(a = 2.8 \text{ m s}^{-2}\). Unfortunately, they do not explain why this value is an upper limit.

An upper limit for the velocity of the trap can be calculated from an adiabacity criterion [11]. For our quadrupole trap we have calculated an average orbital frequency of \(\omega_T/2\pi \approx 40 \text{ Hz}\) for an average orbital radius of \(\rho = 0.9 \text{ mm}\). The maximum velocity for the cloud can then be estimated by

\[
v_{\text{max}} = \frac{2\rho\omega_T}{2\pi},
\]

the velocity of the orbiting atom. It is given by \(v_{\text{max}} = 7.2 \text{ cm s}^{-1}\).

However, I am not too sure about the usefulness of this criterion because it would lead to a too long transfer time.

### 7 Technical data for the coils

In the table below the technical data for all coils used in the transfer system are summarized. The resistance and power is always given for a single coil.

<table>
<thead>
<tr>
<th>Coil</th>
<th>Push</th>
<th>MOT</th>
<th>Horizontal</th>
<th>Vertical</th>
<th>Gradient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inner diameter</td>
<td>38.0 mm</td>
<td>70.0 mm</td>
<td>24.80 mm</td>
<td>50.0 mm</td>
<td>50.0 mm</td>
</tr>
<tr>
<td>Outer diameter</td>
<td>66.6 mm</td>
<td>98.0 mm</td>
<td>60.96 mm</td>
<td>104.3 mm</td>
<td>82.0 mm</td>
</tr>
<tr>
<td># radial layers</td>
<td>10</td>
<td>14</td>
<td>16</td>
<td>19</td>
<td>16</td>
</tr>
<tr>
<td># axial layers</td>
<td>10</td>
<td>3</td>
<td>2</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Resistance</td>
<td>0.088 (\Omega)</td>
<td>0.068 (\Omega)</td>
<td>0.026 (\Omega)</td>
<td>0.098 (\Omega)</td>
<td>0.082 (\Omega)</td>
</tr>
<tr>
<td>Maximum current</td>
<td>20.7 A</td>
<td>46.6 A</td>
<td>90.7 A</td>
<td>42.7 A</td>
<td>42.7 A</td>
</tr>
<tr>
<td>Maximum power</td>
<td>38 W</td>
<td>148 W</td>
<td>214 W</td>
<td>179 W</td>
<td>150 W</td>
</tr>
<tr>
<td>Voltage</td>
<td>1.8 V</td>
<td>3.2 V</td>
<td>2.4 V</td>
<td>4.2 V</td>
<td>3.5 V</td>
</tr>
</tbody>
</table>

Table 2: Technical data for the coils

An important remark concerns the vertical transfer coils: They are consisting of two separate coils that are put in series each with 19 radial loops and two axial layers.

\(^3\)The \textit{jerk} is the third time derivative of the spatial coordinate.
For all coils, the specified dimensions are given. The actual maximum values for the outer diameters of horizontal and vertical transfer tube are \( d_o = 61.4 \) and \( d_v = 106.8 \) respectively.

The dimensions of the copper wires used for the horizontal transfer coils are 1.13 mm \( \times \) 2.63 mm including an insulation of 0.13 mm thickness. For the vertical transfer coils wire with dimensions are 1.43 mm \( \times \) 2.33 mm also including 0.13 mm of insulation. The same wire has been used for the push coil in simulations.

8 Support and cooling system

The support and cooling system for the coils consists of a separable part for the MOT and the horizontal transfer coils and special mounts for the vertical transfer coils.

8.1 Design of the mounts

Figure 9: Horizontal cooling plate
The mounts for the coils have to serve several purposes which are the support and alignment of the coils and their cooling. High stability as well as good cooling properties are provided by a mount made from brass. The brass mounts – brass having a high thermal conductivity – are externally water cooled. A disadvantage of brass is however its high electric conductivity that makes it necessary to actively avoid eddy currents as a source of additional magnetic fields. For that reason, the cooling plates for the vertical transfer are split whereas a backbone made of plastic electrically disconnects the left and right part of the horizontal mounts. This avoids closed current loops in the metal. Another important issue is the alignment of the coils in their mounts. In order align the coils as precise as possible with respect to each other, they are glued into fitting recesses within the brass mount. As a glue, thermal conductive epoxy will be used preserving the good thermal contact between the copper coils and the brass. The epoxy furthermore electrically insulates the coils from each other.
8.2 Assembly of the system

Whereas the mount for the MOT and the horizontal transfer coils can be removed during bake–out of the vacuum system, the vertical transfer coils are fixed once the system is assembled. To bake the vertical transfer tube, thin foil–like heaters\(^4\) will be wrapped around the tube permanently surrounded by heat insulation. While baking the system the coils have to be water cooled to avoid damage on the wire insulation or the epoxy.

When the brass supports including the coils are mounted to the vacuum system for the first time, at the beginning the three upper vertical cooling plates are put around the vertical tube and hold up close to the lattice chamber. Secondly, the MOT coil part of the horizontal cooling plate is set into place. Then the mount for the horizontal transfer coils is put on top and below respectively and both parts are connected after the lowest vertical transfer coil has been placed. Finally, the vertical transfer plates are connected to their spacers. The orientation of the vertical transfer plates is not arbitrary but it is important to line up the slits at one side.

9 Collected material

9.1 Inventor drawings

The table below summarizes all final Inventor drawings of the transfer system together with a short explanation of the part.

9.2 Mathematica code

The table below summarizes the Mathematica code written to simulate the transfer system. In order to study the influence of noise and misalignment the code Horizontal transport will be the most appropriate as each trap is shiftable in all three spatial dimensions. To study the effect of misalignment within a quadrupole trap, meaning a shift between lower and upper coil, the program will have to be modified.

<table>
<thead>
<tr>
<th>File name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assembly_System</td>
<td>Complete experimental set-up including</td>
</tr>
<tr>
<td>Backbone</td>
<td>Plastic connection for the horizontal cooling plate</td>
</tr>
<tr>
<td>coil_grad</td>
<td>Gradient coil</td>
</tr>
<tr>
<td>Coil_plus_housing_vert</td>
<td>Vertical transfer coil in cooling plate</td>
</tr>
<tr>
<td>coil_transfer</td>
<td>Horizontal transfer coil</td>
</tr>
<tr>
<td>coil_transfer_verts</td>
<td>Vertical transfer coil</td>
</tr>
<tr>
<td>Coil_plus_housing_vert_4layer</td>
<td>Final vertical transfer coil in cooling plate</td>
</tr>
<tr>
<td>Connector_MOT_Horiz</td>
<td>Connector for MOT and horizontal cooling plate</td>
</tr>
<tr>
<td>Cooling_Plate_insertion_lower_down</td>
<td>Lower horizontal cooling plate</td>
</tr>
<tr>
<td>Cooling_Plate_insertion_lower_up</td>
<td>Upper horizontal cooling plate</td>
</tr>
<tr>
<td>Cooling_MOT</td>
<td>Cooling plate MOT coil including coil</td>
</tr>
<tr>
<td>Cooling_Plate_complete_down</td>
<td>Complete lower horizontal cooling plate</td>
</tr>
<tr>
<td>Cooling_Plate_complete_up</td>
<td>Complete upper horizontal cooling plate</td>
</tr>
<tr>
<td>CoolingPlate_vert_fit_3</td>
<td>Final vertical cooling plate</td>
</tr>
<tr>
<td>cube_1.33in</td>
<td>Mini cube</td>
</tr>
<tr>
<td>Mount_PushCoil</td>
<td>Suggestion for a push coil mount</td>
</tr>
<tr>
<td>ScienceCell_3_Complete</td>
<td>Lattice chamber including recessed flange</td>
</tr>
<tr>
<td>Spacer_ceramic</td>
<td>Plastic connection for MOT coil cooling plate</td>
</tr>
</tbody>
</table>

Table 3: Inventor drawings of the transfer system
<table>
<thead>
<tr>
<th>File name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal transport</td>
<td>Transfer for horizontal transfer coils</td>
</tr>
<tr>
<td>Transfer from MOT</td>
<td>Includes MOT coils and push coil</td>
</tr>
<tr>
<td>Horizontal-Vertical</td>
<td>Transition from horizontal to vertical transfer</td>
</tr>
<tr>
<td>Vertical transport</td>
<td>Simulation of the whole vertical transfer</td>
</tr>
</tbody>
</table>

Table 4: Mathematica code for the transfer system

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