Diffusive Spin Transport in a Unitary Fermi Gas

by

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Abstract

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This report encapsulates recent work measuring the spin transport properties of strongly interacting fermions. In particular this report presents the first measurement of the transverse spin diffusivity in a unitary Fermi gas in three dimensions finding a result near the naive quantum limit of $\hbar/m$. Further, I present possible extensions of this work to measuring nonlinear deviations from the simplest diffusion model. Here, it is seen that the experiment has the capability to explore the so-called Leggett-Rice effect, thus far only seen weakly interacting fermionic systems.
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Chapter 1

Introduction

1.1 Ultracold Neutral Atoms

Ultracold neutral atoms provide a versatile test bed for condensed matter physics. The primary advantage of studying many-body physics with ultracold gases is the degree to which atomic systems can be controlled. Numerous methods have been established to control the atoms internal and external degrees of freedom. In particular, parameters like the scattering length, Fermi energy, temperature, etc. can be tuned over a wide range unlocking many interesting regimes of study.

While transport measurements are a staple of condensed matter experiments, they are typically only a perturbative departure from equilibrium. In contrast, ultracold atoms present a unique forum in which to study true non-equilibrium many-body dynamics as the timescales can be slow compared to the probes and initialization, yet fast compared to the environmental coupling.

1.2 Universal Properties and Transport

Utilizing a Feshbach scattering resonance one can tune the interactions in a dilute gas of neutral atoms to be as strong as quantum mechanics allows [6]. These so-called unitary
gases - in particular unitary Fermi gases - share universal thermodynamics [18] that determine their properties in equilibrium regardless of the physical origin of the interaction. In this way, quark-gluon plasmas, neutron stars, and ultracold atoms at a scattering resonance share certain characteristics when they are in this strongly interacting regime. These universal properties often relate macroscopic quantities of the system regardless of their microscopic origin and should not depend on the interatomic potentials.

Further, it has been proposed that transport in a unitary Fermi gas also has universal features. In strongly interacting Fermi gases new transport phenomena arise. For instance, near perfect fluidity can occur where the ratio of shear viscosity to entropy density $\eta/s \geq \hbar/4\pi k_B$ is bounded from below by quantum mechanics [8].

The related question - whether quantum mechanics provides a universal bound for spin diffusion - has been recently addressed [14, 20, 25]. This universal behaviour can be estimated on general grounds. At a Feshbach resonance the scattering cross-section, $\sigma$, between atoms of opposite spin is given by the square of the de Broglie wavelength. In the degenerate regime this goes as $\sigma \approx 1/k_F^2$, where $k_F = (6\pi^2 n)^{1/3}$ is the Fermi wavevector and $n$ is the density of atoms in each spin state. The mean free path between collisions is then $l = 1/(n\sigma) \approx 1/k_F$. The average speed, $v$, of atoms is on the order of the Fermi velocity, $\hbar k_F/m$. Therefore, when estimating the diffusivity $D = vl$, the density dependent factors cancel giving $D = \hbar/m$ as a naive limit for the unitary Fermi gas. The scaling argument presented here relies on the assumption that transport is governed by quasi-particles. This quasi-particle picture is known to be violated and breaks down in “bad metals” where the resistivity scales linearly with temperature [9].

1.3 Outline of Thesis

This thesis describes recent efforts to explore the demagnetization dynamics in a degenerate Fermi gas. In particular, it presents a measurement of a spin diffusivity tantalizingly
close to the naive quantum limit, $h/m$. Chapter 2 provides a theoretical overview for the physical models involved in describing the experiment. Further, it discusses experimental signatures indicating the presence of the Leggett-Rice effect. In Chapter 3 the experimental apparatus is described with emphasis on the construction and calibration of new current supplies for fine control over the magnetic field gradients. Chapter 4 summarizes the results appearing in [4]. Chapter 5 discusses the first attempts to observe the Leggett-Rice effect in a strongly interacting ultracold fermionic system. Finally, Chapter 6 provides conclusions and an outlook for future studies.
Chapter 2

Spin Diffusion

2.1 Diffusive Loss of Magnetization

When the magnetization $\mathbf{M}(\mathbf{r},t)$ varies slowly in space, a spin current density $\mathbf{j}_s$ is established to restore global equilibrium. Notice, we adopt the convention that $\mathbf{M}$ represents a vector in spin space, while $\mathbf{r}$ represents a vector in real space. Throughout this discussion we utilize a Bloch sphere picture where the magnetization is defined as $\mathbf{M}=(M_\perp \cos \phi, M_\perp \sin \phi, M_z)$ shown in Figure 2.1.

We expect a contribution to the rate of change of $\mathbf{M}(\mathbf{r},t)$ from three effects [22]. First, we anticipate precession around the local external field $\mathbf{H}(\mathbf{r})$. The second contribution is decay due to spin-nonconserving interactions. Lastly, we expect a contribution from spin diffusion.

The contribution from the external field is described by the usual torque term

$$\left[ \frac{\partial \mathbf{M}}{\partial t}(\mathbf{r},t) \right]_{\text{ext}} = \gamma \mathbf{M} \times \mathbf{H}(\mathbf{r}) \equiv \mathbf{M} \times \Omega(\mathbf{r})$$

(2.1)

where $\gamma$ is the gyromagnetic ratio. The second effect is typically described by relaxation times $T_1$ and $T_2$. In the cases considered here these relaxation times are much longer than the duration of the experiment and so their effect on the decay of magnetization can be ignored.
Chapter 2. Spin Diffusion

Figure 2.1: The Bloch sphere picture showing the two interacting spin states, $|\uparrow\rangle$ and $|\downarrow\rangle$, we consider. The magnetization is represented as a vector on the Bloch sphere, broken down into its component along the $z$ direction, $M_z$, and the component perpendicular, $M_\perp$.

The last effect, diffusion, is typically represented by

$$\left[ \frac{\partial M}{\partial t}(\vec{r}, t) \right]_{\text{diff}} = D \nabla^2 M(\vec{r}, t)$$  \hspace{1cm} (2.2)

where $D$ is the spin diffusivity. However, it has been shown that for an interacting Fermi liquid this diffusion term must be modified [21]. The basis for the modifications provided in [21] is the fact that in an interacting Fermi liquid any polarization of the spins will give rise to a molecular field, and any given spin will then see (and precess around) a total field that is the sum of the external and molecular fields. This molecular field can cause additional precession of the spin current, which effects the relaxation of $M(\vec{r}, t)$.

As a result the equation describing spin diffusion is given by [21]

$$\frac{\partial M(\vec{r}, t)}{\partial t} - M \times \Omega(\vec{r}) = D \frac{\partial}{\partial z} \left[ \frac{1}{1 + \mu^2 M^2} \left( \frac{\partial M}{\partial z} + \mu \left( M \times \frac{\partial M}{\partial z} \right) + \mu^2 \left( M \cdot \frac{\partial M}{\partial z} \right) M \right) \right].$$  \hspace{1cm} (2.3)

We only consider linear magnetic field gradients $\vec{B}(\vec{r}) = \vec{B}_0 + \vec{B} z$ and without loss of generality define the coordinate system such that the gradient is always in the $z$ direction.
Figure 2.2: Longitudinal diffusion is driven by a gradient in the amplitude of magnetization, while transverse diffusion is driven by a gradient in the direction of the magnetization.

From equation (2.3) it can be seen that spin diffusion is driven by either a gradient in the amplitude of magnetization or its direction as illustrated in Figure 2.2. When the gradient, \( \partial \textbf{M} / \partial z \), is parallel to \( \textbf{M} \), the right-hand side of equation (2.3) reduces to (2.2). This leads to longitudinal spin diffusion. Conversely, when the gradient is orthogonal to \( \textbf{M} \) this leads to transverse spin diffusion. In this case, the amount to which the diffusion term in (2.3) is modified from (2.2) is quantified by \( \mu \textbf{M} \).

We first consider the simplest case where \( \mu = 0 \) and the diffusion term is not modified from (2.2). In this case the diffusion equation reduces to

\[
\frac{\partial \textbf{M}}{\partial t} = \textbf{M} \times \Omega + D \partial^2_z \textbf{M}. \tag{2.4}
\]

Here we assume \( M_z \) and \( |M_\perp| \) to be homogeneous at all times and therefore consider the dynamics of the transverse magnetization, \( M_\perp = M_x + iM_y \), subject to transverse spin diffusion with diffusivity \( D_\perp \).

When considering the transverse magnetization and a linear magnetic field gradient in the \( z \) direction one can rewrite equation (2.4) as

\[
\frac{\partial M_\perp}{\partial t} = -i\alpha z M_\perp + D_\perp \partial^2 z M_\perp \tag{2.5}
\]

where \( \alpha = \Delta \mu B'/\hbar \), \( \Delta \mu \) is the difference in magnetic moment between \( |\uparrow\rangle \) and \( |\downarrow\rangle \), and \( B' \) is the magnetic field gradient.
Given a homogeneous initial magnetization, $M_\perp(\vec{r}, t = 0) = \text{const}$, atoms are non-interacting due to the Pauli exclusion principle. Further, our fermionic atoms have no intrinsic mechanism for magnetization to decay. However, given the different magnetic moments of the two internal states, a gradient in magnetic field will twist the magnetization into a spiral pattern leading to a gradient in transverse magnetization. Diffusive spin currents then drive spins to move and collide allowing spin diffusion to erode the magnetization amplitude.

After a time $t$, the amplitude of the magnetization is described by a solution to equation (2.5) \cite{1}

$$M_\perp(\vec{r}, t) = i \exp[-i\alpha z t - t^3/6\tau^3]$$ \hspace{1cm} (2.6)

where

$$\tau = (2D_\perp \alpha^2)^{-1/3}. \hspace{1cm} (2.7)$$

This solution also arises as a limiting case of equation (2.3) and will be discussed in detail later in this report.

If a spin refocussing $\pi$ pulse is applied at time $t_\pi$, the spiral in magnetization will untwist at time $t = 2t_\pi$ resulting in a uniform magnetization that decays in amplitude with time \cite{26}

$$M_\perp(\vec{r}, t) = -i \exp[-t^3/24\tau^3].$$ \hspace{1cm} (2.8)

To measure $M_\perp$ we perform a simple Ramsey experiment. This experiment occurs in a two level system ($|\uparrow\rangle$, $|\downarrow\rangle$) coupled by resonant radio frequency (r.f.) pulses. In this type of experiment we begin with the magnetization aligned along the external field which we take to be the $z$ axis (spin polarized in state $|\uparrow\rangle$). The first r.f. pulse rotates the magnetization by an angle $\pi/2$ around the $x$ axis creating an initial magnetization $M_\perp = i$. The subsequent $\pi$ pulse rotates the magnetization by $180^\circ$, again around the $x$ axis. Lastly, the final $\pi/2$ pulse projects the transverse magnetization onto the $z$ axis. Varying the phase, $\psi$, of the final $\pi/2$ pulse, with respect to the first two pulses, allows
one to probe the phase accumulated. The fraction of atoms in $|\downarrow\rangle$ then varies as a function of the phase of the last pulse. The fringe measured at each hold time,

$$P_\downarrow(\psi) = \frac{A(t)}{2} \sin(\theta(t) - \psi) + \frac{1}{2}$$

(2.9)

where $\psi$ is the phase of the last $\pi/2$ pulse, provides a measure of $M_\perp(t) = A(t) \exp[i\theta(t)]$ and allows for the extraction of both the amplitude of the magnetization, $A(t)$, and the accumulated phase, $\theta(t)$.

In this discussion we have assumed that the diffusivity is simply a constant. In reality, this coefficient depends on a number of parameters [14]. In general, $D_\perp$ depends on temperature and polarization. In liquid helium experiments [2] these parameters can be controlled and kept constant quite easily. However, in ultracold atomic systems the temperature and polarization evolve dynamically throughout the experiment complicating the measurement of diffusivity. This will be discussed in more detail in Chapter 4.

### 2.2 Leggett-Rice Effect in Spin Echo Experiments

For a nonzero $\mu$ the form of (2.3) is complicated and nonlinear. As a result, the relaxation of the magnetization has complex behaviour that depends not only on the temperature and external field but also on the initial pulse angle. In this section, we explore the regime in which a measurement of this quantity is feasible and the possible experimental signatures of the Leggett-Rice effect.

We apply equation (2.3) to a Ramsey type experiment we will call “$\phi - \pi - \pi/2$”, illustrated in Figure 2.3. To be sensitive to $\mu$ we replace the first $\pi/2$ pulse in a typical Ramsey experiment with an r.f. pulse that has a variable pulse area $\phi$.

In this type of experiment we assume that the component of $M$ in the $xy$ plane is spatially varying in direction but constant in amplitude leading to transverse spin diffusion. With these assumptions, the last term on the right hand side of (2.3) vanishes.
Chapter 2. Spin Diffusion

Figure 2.3: The pulse sequence to explore the Leggett-Rice effect in a spin-1/2 system. The initial pulse rotates the Bloch vector by an angle $\phi$ around the $x$ axis. The subsequent $\pi$ pulse rotates the Bloch vector by 180° around the $x$ axis. The last pulse projects the Bloch vector onto the $z$ axis. Depending on the relative phase of the last pulse, the Bloch vector will be rotated creating a Ramsey interference fringe.

Further, since $M^2$ is constant in space, (2.3) reduces to

$$\frac{\partial M}{\partial t} - M \times \Omega(\vec{r}) = \frac{D}{1 + \mu^2 M^2} \left[ \partial^2_z M + \mu \left( M \times \partial^2_z M \right) \right].$$ (2.10)

Decomposing the magnetization into components in spin space gives

$$\begin{pmatrix} \dot{M}_x - M_y \Omega \\
\dot{M}_y + M_x \Omega \\
\dot{M}_z \end{pmatrix} = \frac{D}{1 + \mu^2 M^2} \begin{pmatrix} M''_x + \mu(M_y M''_z - M_z M''_y) \\
M''_y + \mu(M_z M''_x - M_x M''_z) \\
M''_z + \mu(M_x M''_y - M_y M''_x) \end{pmatrix}$$ (2.11)

where $M''_i$ represents the second partial derivative with respect to $z$ of the $i$-th spin component of the magnetization and $\dot{M}_i$ represents the partial derivative with respect to time of the $i$-th spin component of magnetization.

For the case we consider with no field curvature and a homogeneous initial state, $\dot{M}_z = 0$. Therefore, we again consider just the transverse magnetization $M_\perp = M_x + iM_y$ which reduces equation (2.10) to

$$\dot{M}_\perp + i\Omega M_\perp = \frac{D\perp (1 - i\mu M_z)}{1 + \mu^2 M_\perp^2 + \mu^2 M^2_\perp} M''_\perp.$$ (2.12)

At this point we introduce the ansatz in [21] for the solution of equation (2.12)

$$M_\perp(\vec{r}, t) = A(t) \exp [i\theta(t)] \exp [-i\xi(\vec{r}, t)]$$ (2.13)
where $\xi(\vec{r}, t)$ is constructed to incorporate the ordinary precession due to the external field and all effects of the echo pulses. This means that any intrinsic phase lag due to the additional diffusion term quantified by $\mu M$ is captured by $\theta(t)$. Substituting (2.13) into (2.12) gives equations for $A(t)$ and $\theta(t)$

\[
\frac{\partial A(t)}{\partial t} = \frac{D^\perp A(t)(\xi')^2}{1 + \mu^2 M_z^2 + \mu^2 A^2(t)} \quad (2.14)
\]

\[
\frac{\partial \theta(t)}{\partial t} = -\frac{D^\perp \mu M_z (\xi')^2}{1 + \mu^2 M_z^2 + \mu^2 A^2(t)}. \quad (2.15)
\]

### 2.2.1 Solutions and Limiting Cases

Equations (2.14) and (2.15) are integrable and can easily be solved. Noticing that (2.15) can be written as

\[
\frac{\partial \theta(t)}{\partial t} = \frac{\mu M_z}{A(t)} \frac{\partial A(t)}{\partial t} \quad (2.16)
\]

the solution is

\[
\theta(t) - \theta(t_0) = \mu M_z \ln \left[ \frac{A(t)}{A(0)} \right] \quad (2.17)
\]

which still requires the solution for $A(t)$. Integrating equation (2.14) gives

\[
(1 + \mu^2 M_z^2) \ln \left[ \frac{A(t)}{A(0)} \right] - \frac{1}{2\mu^2} [A(0)^2 - A(t)^2] = -D^\perp \int_{t_0}^{t} (\xi')^2(t') \, dt'. \quad (2.18)
\]

To further simplify this equation we utilize the form of the twist imparted to the magnetization from the gradient, $\xi' = \alpha t$. Recalling the result (2.7) then gives

\[
(1 + \mu^2 M_z^2) \ln \left[ \frac{A(t)}{A(0)} \right] - \frac{1}{2\mu^2} [A(0)^2 - A(t)^2] = -\frac{t^3}{24\tau^3}. \quad (2.19)
\]

for the amplitude at times $t = 2t_\pi$.

In the case where $\mu = 0$ the solution (2.8) is restored. Similarly, when $M_z \to 1$ we can neglect the second term on the left-hand side of equation (2.19). This is also satisfied in the limit where $A(t) - A(0) \ll 1$ or $\mu \ll 1$. Working within these limits we arrive at the same exponential as in (2.8), however, with an effective diffusivity $D_{\text{eff}} = D^\perp/(1 + \mu^2 M_z^2)$. 

Figure 2.4: Leggett-Rice effect for $\mu = 1.0$. The solid red line shows the limit as $M_z \to 1$ with effective diffusivity $D_{\text{eff}}$. The dashed red line plots equation (2.8). The curves between these two limits show the numerical solution of (2.18) for different mixing angles $\phi = \pi/16, 2\pi/16, ..., 7\pi/16$ (light gray to dark gray).

In the case where $M_z = 0$ (as in Chapter 4), the solution to equation (2.19) can be expressed as $W \exp[W] = z$ and is solved by a product logarithm $W(z)$:

$$A(t) = \frac{1}{\mu} \sqrt{W \left( \mu^2 e^{\mu^2 - \frac{1}{\pi} \left( \frac{t}{\tau} \right)^3} \right)}.$$  \hspace{1cm} (2.20)

Between these limiting cases the solution to (2.19) can be found numerically.

In our experiment we can probe the relaxation of magnetization via a spin echo technique similar to the method described in [2]. Here, we follow the amplitude of the magnetization as it relaxes from fully magnetized, $M_\perp = i \sin[\phi]$, to the equilibrium value $M_\perp = 0$.

The relaxation of magnetization for $\mu = 1$ is shown in Figure 2.4. Here all the data is normalized by the initial magnetization causing the curves to collapse on top of each other for short times. However, the curvature for long times is seen to change significantly over this range of mixing angles. The change in curvature corresponds to a change in the effective decay time of the magnetization and therefore a change in the apparent diffusivity via equation (2.7). Resolving this variation in the diffusivity may be difficult in our experiments as it requires excellent signal to noise for long hold times. This will
Figure 2.5: Leggett-Rice effect for $M_z = 0$ and varying the spin rotation parameter from $\mu = 0$ (dashed red line) through $\mu = 0.25, 0.5, ..., 4$ (light gray to dark gray).

be discussed in the context of our measurements in Chapter 5.

The impact of the Leggett-Rice effect on $|M_\perp(t)|$ can be seen when beginning the spin echo sequence with $\phi = \pi/2$. As in Figure 2.5, a larger $\mu$ increases the apparent magnetization decay time. In a different framework, the relaxation of magnetization can be seen as a loss of coherence in the sample. Then the preservation of the initial magnetization can be linked to an increased coherence time. This effect has been observed in bosons [12] and could be accessed in our experiment by tuning the scattering length to tune the spin rotation parameter. The connection between the spin rotation parameter discussed in [12] and the Leggett-Rice effect analyzed here is discussed extensively in [24] and the two become analogous in the limit of a Boltzmann gas.

By increasing $\mu$, one can enter a regime where a spontaneous rephasing of the spins occurs as they precess around the molecular field. This is illustrated in Figure 2.6 where the spin rotation effect acts as an analogue to a spin echo. Two colliding spins rotate around the net spin. When the rotation reaches $\pi$ an effective local spin-echo has occurred that will allow the spins to rephase, thus increasing the coherence time in the sample and increasing the time the sample remains magnetized.

While our experiment is sensitive to the amplitude of the magnetization, the Ramsey
Figure 2.6: Two interacting spins precess at different frequencies. When they collide they rotate about the net spin vector. If the rotation approaches $\pi$ a local spin-echo has occurred and the spins can rephase, increasing the coherence time.

Figure 2.7: The growth of the phase for $\mu = 1.0$ from $\phi = \pi/16, 2\pi/16, ..., 7\pi/16$ (light gray to dark gray) and $M_z = 1$ (red line).

...fringes also provide a measure of the accumulated phase. Following the solution (2.17), the Leggett-Rice effect could also be observed in our experiment as a shift in the phase measured. This is shown in Figure 2.7 for a variety of mixing angles. In all cases, the growth in the phase is approximately cubic. As an example, at an amplitude of $A(t) = 0.5$, $M_z = 0.5$, and $\mu = 0.5$ the accumulated phase shift would be $\theta(t) \approx 25^\circ$.

This type of phase shift is within the experimental resolution as is discussed in Chapter 5.

Equations (2.17) and (2.18) provide an excellent starting point from which we can
begin to explore nonlinear corrections to the diffusion term (2.2). The technique of using Ramsey fringes to measure the amplitude and phase for a given hold time provides an excellent avenue to quantify any deviations from the simplistic model used for $\mu = 0$. 
Chapter 3

Experimental Apparatus

In this experiment we utilize the dual species Rb-K machine described in [3]. We begin each experimental cycle with a dilute vapour of $^{40}$K and $^{87}$Rb in a single vapour cell. Six counter-propagating laser beams trap and cool $\sim 1 \times 10^9$ Rb atoms and $\sim 2 \times 10^7$ K atoms in a magneto-optical trap (MOT). These atoms are then magnetically transferred to the nearby “chip trap” where they are trapped in a magnetic field minimum $\sim 180\mu$m from the chip. In this stage, Rb is evaporated directly, sympathetically cooling K. Subsequently, both species are transferred to a crossed optical dipole trap (ODT) where a second stage of evaporation occurs. Lastly, a resonant light pulse removes the remaining Rb, leaving $\sim 3 \times 10^4$ spin polarized $^{40}$K atoms in the $|F = 9/2, m_F = -9/2\rangle$ state at a temperature of $T \approx 0.25T_F$ where $T_F$ is the Fermi temperature. The ODT has final trapping frequencies $(\omega_x, \omega_y, \omega_z) = 2\pi \times (191,786,786)\text{Hz}$. Note, that these experiments are carried out at a high magnetic field where $F$ and $m_F$ are no longer good quantum numbers. However, for convenient labelling we will refer to these states via the quantum numbers $F$ and $m_F$ that label them at low field.
3.1 Gradient Control and Compensation

We implement gradient control via micro-fabricated wires on the nearby atom chip. During the Feshbach stage of the experiment the MOT coils are placed into Helmholtz configuration to obtain a uniform, gradient-free magnetic field at the center of the coils near 202.1 G. However, the atom chip/ODT sits \( \sim 25 \) mm above center of the Feshbach/MOT coils as in Figure 3.1, causing a residual gradient of approximately 10G/cm in the vertical direction. A complete characterization of the MOT coils is given in [16] and a characterization of the homogeneity of the magnetic field at the ODT position is given in [10]. Excepting the large vertical magnetic field gradient induced by the offset of the ODT from the center of the coils, the residual curvatures in the field are negligible (4mG/mm\(^2\)). This leaves only linear magnetic field gradients to consider.

The experiment could be carried out with the magnetic field gradient caused by the offset from the center of the MOT coils. However, given the form of equation (2.7) it is desirable to be able to arbitrarily control the gradient in order to measure the diffusivity. Fortunately, the proximity of the ODT to the chip trap provides a number of options to control the magnetic field gradient.

In order to compensate and arbitrarily control the magnetic field gradient at the position of the ODT we utilize two nearby chip wires. The geometry of the chip wires and field compensation is shown in Figure 3.2. In order to create horizontal and vertical magnetic field gradients (\( \propto I_1 \mp I_2 \)) we apply a small current through two parallel chip wires with opposite polarity. Due to the geometry, fixing the sum of the two currents \( (I_1 + I_2) \) provides a constant bias field that compensates the vertical field gradient while varying the difference of the two currents \( (I_1 - I_2) \) allows for the tuning of the gradient along the Feshbach field direction.

The small controlled currents used to set the gradient in our experiment are provided by two bipolar current supplies constructed in the lab. The measured characteristics and a general schematic of the current supply can be found in [23]. These supplies provide
Figure 3.1: A schematic of the MOT coils from [16]. The ODT position (orange circle) is located above the center of the MOT coils. This offset from the center of the coils results in a magnetic field gradient of $\sim 10 \text{ G/cm}$. 
Figure 3.2: The magnetic field gradient in the $y$ direction at the position of the atoms can be compensated by fixing the sum ($I_1 + I_2$) of the currents through the two chip wires. The resulting magnetic field in the $z$ direction from the difference ($I_1 - I_2$) of the currents adds in the direction of the Feshbach field.

currents up to roughly 500 mA and can be ramped on or off quickly ($\sim 20\mu s$). The control voltage to current ratio for the current supplies is 10:1, $V_{1,2} = I_{1,2} \times 10\Omega$. Further, the control voltages for the two current supplies are calibrated such that they can be operated in a symmetric configuration where they supply the same current. This corresponds to $V_2 = 1.04V_1$.

3.1.1 Gradient Measurements

To assess the performance and stability of the gradient compensation, we spectroscopically measure the atomic resonance of the $|\frac{9}{2}, -\frac{9}{2}\rangle \rightarrow |\frac{9}{2}, -\frac{7}{2}\rangle$ transition as a function of the applied control voltage and position of the atomic cloud within the magnetic field.

To determine the required voltage sum to cancel the vertical field gradient we first prepare a polarized gas in the $|\frac{9}{2}, -\frac{9}{2}\rangle$ state. We then ramp the gradient compensation on in approximately 5ms. Lastly, we move the cloud in the ODT to a new vertical posi-
Figure 3.3: **a. Sample spectra:** Three spectra taken for a voltage sum of 0.6 V. The three data sets are for piezo control voltages of 0, 60, and 120 V (circles, squares, triangles). The lines are Lorenztian fits to each data set. **b. Gradient versus voltage sum:** The data points are extracted from the linear fits in the inset of this figure. A linear fit, \( y = mx + b \), finds \( m = 28.60 \pm 0.62 \) and \( b = -12.84 \pm 0.50 \).

We extract the resonance frequency as a function of the cloud position and, using a linear least squares fit, find \( \frac{\partial}{\partial y} \nu_{res} / \text{cm} \) as a function of voltage sum and then use a Breit-Rabi calculation to convert this resonance frequency into magnetic field as in Figure 3.3b. Finding the \( x \)-intercept of the curve, 0.458 V, allows for the cancellation of the vertical field gradient.

Having found the voltage sum required to cancel the vertical field gradient we work at
Figure 3.4: **a. Sample spectra:** Three spectra taken for a voltage difference of -0.7 V. The three data sets are for piezo control voltages of 0, 60, and 120 V (circles, squares, triangles). The lines are Lorentzian fits to each data set. **b. Gradient versus voltage difference:** The data points are extracted from the linear fits in the inset of this figure. A linear fit, \( y = mx + b \), finds \( m = -34.31 \pm 1.02 \) and \( b = -0.43 \pm 0.48 \).

This value and vary the voltage difference to measure the shift in the resonance frequency in the \( z \) direction, \( \partial_z \nu_{\text{res}} \). Following the same procedure described above, we move the cloud in the \( z \) direction with piezo controlled mirrors and measure the resonance frequency as a function of position in the magnetic field gradient (Figure 3.4).

Here, the piezo control allows for an adjustment of \((0.218 \pm 0.010) \mu m/V_{\text{piezo}}\) or a total range of \((26.16 \pm 1.20) \mu m\). In this calibration, we roughly determine the gradient we induce for various voltage differences. The statistical uncertainty is small as the quality of Lorentzian fits to the r.f. spectra are quite good. Further, the stability of the external Feshbach field provides a systematic uncertainty in the the position of the resonance of roughly 3 kHz, on the order of the linewidth found in the fits.
Figure 3.5: The data is extracted from Lorentzian fits to spectra sampled over a range of voltage differences and ODT positions. The final linear fit, \( y = mx + b \), finds \( m = 2.20 \pm 0.03 \) and \( b = -1.46 \pm 0.02 \).

Figure 3.6: Absolute gradient in the \( z \) direction versus voltage difference combing both the gradient induced in the \( z \) and \( x \) directions.

Lastly, we check any gradients introduced in the \( x \) direction by repeating this procedure a final time and moving the cloud in the \( x \) direction. We find a small contribution to the net gradient as shown in Figure 3.5.

Combining the results for the \( x \) gradient and \( z \) gradient as a function of voltage difference allows for a full calibration of the gradient we can introduce at a voltage sum of 0.458 V. The absolute gradient is shown in Figure 3.6.

The shaded region of this plot represents the systematic uncertainty in the gradient calibration. For a small voltage difference, the linewidth we measure is quite narrow.
Figure 3.7: The state selective imaging procedure. Atoms in $|9/2, -5, 2\rangle$ are mapped to $|9/2, -9/2\rangle$. A Stern-Gerlach pulse correlates the atomic state to the final atom position allowing us to count the relative number of atoms in two internal states.

However, as we increase the voltage difference the linewidth increases and sets the uncertainty in our measurement. It is also interesting to note that the voltage difference for zero gradient is offset from zero voltage difference. This is due in part to the contribution from the $x$ gradient in the calibration. Figure 3.6 provides the reference for all gradient values presented in G/cm.

### 3.2 State Selective Imaging

Our imaging scheme allows us to simultaneously count the populations of atoms in the $|9/2, -9, 2\rangle$ and $|9/2, -5/2\rangle$ states. The ability to concurrently measure two spin states is ideally suited for our r.f. spectroscopy measurements. The typical imaging scheme maps the original population in $|9/2, -5/2\rangle$ to $|9/2, -9/2\rangle$ while leaving the original $|9/2, -9/2\rangle$ population relatively unaffected as shown in Figure 3.7. We then perform resonant absorption imaging on the $|9/2, -9/2\rangle \rightarrow |11/2, -11/2\rangle$ cycling transition to observe both populations.

During the typical experimental cycle, the main experiment takes place at the Feshbach resonance located at $202.10(3)$ G. Prior to imaging we quickly jump the magnetic field ($20\mu s$ step followed by $\sim 5$ms settling time) to the zero crossing of the Feshbach
resonance at 209 G as described in [10]. We also utilize the settling time to ramp off the gradient compensation. We image at the zero-crossing of the s-wave resonance to minimize interactions and loss during time of flight (TOF).

Once the magnetic field has rung down following the field jump we turn off the ODT and release the atoms for TOF expansion. Immediately after releasing the atoms from the optical trap, we apply a short pulse of a strong magnetic field gradient to give a differential momentum kick to each of the three states. This short Stern-Gerlach pulse correlates the position of the atoms in the eventual absorption image with the initial atomic state. Further, during TOF, the atoms fall in a weak magnetic field gradient allowing us to resolve r.f. transitions between spatially separated magnetic states.

Directly after the Stern-Gerlach pulse, before the spin states have separated, we apply a \( \pi \) pulse resonant with the \( | \frac{9}{2}, -\frac{7}{2} \rangle \rightarrow | \frac{9}{2}, -\frac{5}{2} \rangle \) transition. This r.f. pulse switches the populations of these two states. After a short TOF, during which the spin states separate spatially, we apply another \( \pi \) pulse, this time resonant with the \( | \frac{9}{2}, -\frac{9}{2} \rangle \rightarrow | \frac{9}{2}, -\frac{7}{2} \rangle \) transition. The frequency of this r.f. pulse is tuned such that it is on resonance for the atoms originally in the \( | \frac{9}{2}, -\frac{5}{2} \rangle \) state but off-resonant for the atoms originally in the \( | \frac{9}{2}, -\frac{9}{2} \rangle \) state because of their relative positions in the weak magnetic field gradient generated by the Feshbach coils. Therefore, this pulse maps the population of the now \( | \frac{9}{2}, -\frac{7}{2} \rangle \) state to the \( | \frac{9}{2}, -\frac{9}{2} \rangle \) state while only transferring a small fraction of the atoms originally in \( | \frac{9}{2}, -\frac{9}{2} \rangle \) to \( | \frac{9}{2}, -\frac{7}{2} \rangle \). After another short TOF we perform resonant absorption imaging to reveal two spatially separated clouds corresponding to the atoms originally in \( | \frac{9}{2}, -\frac{9}{2} \rangle \) and \( | \frac{9}{2}, -\frac{5}{2} \rangle \).

### 3.2.1 Imaging and Number Calibration

While the state selective imaging scheme allows us to successfully image multiple magnetic states it also impacts our ability to successfully resolve the atom number in each state. Applying the Stern-Gerlach pulse results in an optical density that leads to un-
dercounting of the relative populations. Further, the fraction of $|9/2, -9/2\rangle$ atoms transferred during the final $\pi$ pulse is less than 1 and needs to be accounted for.

To calibrate the fraction of the total measured atom number as a result of the increased optical density we obtain a saturation parameter used to scale the atom number. To carry out this calibration we image at high field for long (5.5 ms) TOF for various levels of imaging light intensity. In this way we obtain a curve for counted atom number versus imaging intensity. Extrapolating this curve back to zero imaging intensity provides us with a calibration of the number imaged at our intensity, $N = (1/1.4)N(0)$.

We calibrate the fraction of atoms inadvertently transferred out of the $|9/2, -9/2\rangle$ state in a simple experiment. We first create a 50/50 mixture of atoms in $|9/2, -9/2\rangle$ and $|9/2, -7/2\rangle$ where we assume that $N_{-9/2} = N_{-7/2}$. Subsequently, we map the atoms in the $|9/2, -7/2\rangle$ state to the $|9/2, -5/2\rangle$ via adiabatic rapid passage and carry out our usual state selective imaging procedure. Imaging both states in this way implies $N_{-5/2} = \beta N_{-9/2}$, where $\beta$ is some numerical factor to account for atoms inadvertently transferred during the imaging protocol. For the data presented in Chapter 4, $\beta \approx 1.3$. 
Chapter 4

Diffusive Spin Transport

Spin diffusion is one of the basic transport processes which tends to compensate an imbalance of magnetization between regions of a sample. It has been studied in various environments, e.g. liquid helium [21, 19], spintronics [27], and ultracold atomic gases [25, 20]. Here we study the spin transport in a unitary Fermi gas. While the thermodynamics of unitary Fermi gases are known to be universal [18], the universality of transport properties is only conjectured.

The conceptual challenge is that quasi-particles are short-lived, confounding the standard paradigm of transport kinetics. For most Fermi-degenerate systems diffusivity is $\tau_s E_F / m$ where $\tau_s$ is the transport lifetime. However, if we consider $\tau_s$ to be the lifetime of some quasi-particle, the Heisenberg uncertainty principle requires that $\tau_s E_F \geq \hbar$ and therefore $D \geq \hbar / m$. In the unitary limit of maximal scattering cross-section, this bound would saturate, giving $D \to \hbar / m$. In this way, spin transport is predicted to be suppressed and reach a unitary limit due to strong scattering.

This chapter expounds upon the results in [4] and clarifies the methods employed in that work.
Figure 4.1: A cartoon of the mechanisms that drive the relaxation of magnetization. A linear gradient twists the homogenous magnetization into a spin spiral that allows diffusive spin currents to restore a global equilibrium.

4.1 Relaxation of Magnetization

Spin diffusion is driven by a gradient in either the amplitude of magnetization or its direction. In this work, we study the relaxation of the transverse magnetization to probe transverse spin diffusion in a unitary Fermi gas. Borrowing from NMR studies of liquids and solids, we study transverse spin diffusion via a spin-echo sequence as in Figure 4.1.

We begin with a spin polarized gas of $^{40}$K in $|9/2, -9/2\rangle$ (labeled $|\uparrow\rangle$) sympathetically cooled to $T \approx 0.25T_F$, where $T_F$ is the Fermi temperature. We apply a $\pi/2$ pulse resonant with the $|9/2, -9/2\rangle \rightarrow |9/2, -7/2\rangle$ (labeled $|\downarrow\rangle$) transition to create an equal superposition of $|\uparrow\rangle$ and $|\downarrow\rangle$ with homogeneous magnetization $M_y = 1$.

A smooth spiral of magnetization is created by a controlled magnetic field gradient, $B' = \partial_z B$, where $B$ is the magnetic field. After some hold time, $t_\pi$, a spin-refocussing $\pi$ pulse switches the populations of $|\uparrow\rangle$ and $|\downarrow\rangle$, causing the spiral of magnetization to untwist at time $2t_\pi$. This leaves a homogeneous but reduced magnetization given by equation (2.8). Varying the phase, $\psi$, of a final $\pi/2$ pulse at time $2t_\pi$ produces interference fringes, from which we can extract the magnetization amplitude and phase as in equation (2.9). To count the populations in both of these states we first map the atoms in state $|\downarrow\rangle$ to $|9/2, -5/2\rangle$ via adiabatic rapid passage and then carry out the imaging procedure.
Figure 4.2: a. Phase coherent Ramsey fringes: A sample Ramsey fringe taken at $\sim 4.72$ G/cm after 0.26 ms hold time. The phase coherence in the sample allows for a good fit to equation (2.9). b. Scrambled phase Ramsey fringes: A Ramsey fringe taken at $\sim 4.72$ G/cm after 6.02 ms where the phases are randomized.

described in Chapter 3. Extracting the amplitude over a range of hold times allows us to follow the relaxation of the magnetization driven by diffusive spin transport.

4.1.1 Ramsey Interference Fringes

Magnetization is measured using a standard $\pi/2-\pi-\pi/2$ Ramsey sequence. The phase of the first and second r.f. pulses are kept constant, while the final pulse has a variable relative phase, $\psi \in [-\pi, \pi)$. The final pulse maps the transverse magnetization onto the quantization axis and is sensitive to the accumulated phase during the relaxation time.

For each hold time, $2t_\pi$, we plot the fraction of atoms in $|\downarrow\rangle$ as a function of the phase of the last $\pi/2$ pulse to resolve interference fringes fit to equation (2.9). For short hold times we find good phase coherence of the sample; enough to resolve reproducible fringes that can be fit to extract the amplitude (Figure 4.2a.). However, for long hold times ($t \geq 1.5$ms) the field stability is insufficient to preserve a reproducible phase throughout the sequence resulting in a randomized phase. These data sets are poorly fit with equation (2.9), as in Figure 4.2b., causing an underestimate of the amplitude of the magnetization.
In order to avoid this underestimation we utilize the standard deviation of the transferred fraction to measure $|M_\perp|$. The standard deviation gives a measure of the spread in the transferred fraction for all the phases sampled and is calculated as

$$
\sigma(x) = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \mu(x))^2}
$$

(4.1)

where $\{x_i\}$ is the data set and $\mu(x)$ is the mean of the data. The calculated standard deviation is normalized by the standard deviation of a data set, $\{y_i\}$, generated by the cosine of the phases sampled, $y_i = B \cos \psi_i$, where $\{\psi_i\}$ is the set of phases sampled and $B$ is the amplitude.

Utilizing the standard deviation as the method to calculate the amplitude of the magnetization provides a consistent estimate for all hold times. A comparison of the fitted amplitude and the standard deviation is given in Figure 4.3. This figure clearly shows that the amplitude from a fit of equation (2.9) to the data is consistently lower than the amplitude calculated from the standard deviation. These two methods of measuring the amplitude find suitable results but, as noted, the calculation of the standard deviation provides a more consistent measurement for all hold times.
Figure 4.4: Exponents $\eta$ extracted from a fit, $y = a \exp \left[ -\left( t/\tau_M \right)^\eta \right] + b$ to the data over a range of gradients.

Additionally, the Ramsey fringes allow us to extract the phase accumulated for any transversely magnetized state, $M_\perp e^{i\theta(t)}$. We do not expect to observe the Leggett-Rice effect in the phase measured for $M_z = 0$ given the form of equation (2.17). However, the phase measured from these Ramsey fringes provides a baseline for the phase noise within which we could resolve an effect due to spin rotation. This data also provides a useful diagnostic for transient effects. This will be discussed in more detail in the context of the Leggett-Rice effect in Chapter 5.

4.2 Transverse Spin Diffusion

Having established a method to dynamically follow the relaxation of magnetization using a Ramsey spin-echo sequence, we apply the model (2.8) to extract time constants over a range of gradients. We fit the measured $|M_\perp|$ to a function of the form $y = a \exp \left[ -\left( t/\tau_M \right)^\eta \right] + b$ where the time constant, $\tau_M$, and the exponent, $\eta$, are left as free parameters.

We notice that over the range of gradients explored, the exponent is consistent with $\eta = 3$ as shown in Figure 4.4. The scatter in the exponent could arise from the Leggett-
Figure 4.5: Magnetization decay times, $\tau$, at various gradients. The horizontal and vertical error bars represent the systematic error in the gradient calibration and the fit error respectively. The solid line is a single parameter fit of equation (2.7) to the data. The inset shows a sample of $|M_\perp|$ at a gradient of $\sim 7.6$ G/cm. The solid line is a fit using the simple diffusion model (2.8).

Rice effect and will be discussed in Chapter 5. However, $\eta = 3$ is not statistically excluded in the data and so we fix the exponent reducing the model to the case in equation (2.8) which assumes $\mu = 0$.

Fixing $\eta = 3$ we refit the data and extract $\tau_M$. Comparing with equation (2.8) we see that $\tau = \tau_M/24^{1/3}$. Rescaling the fitted $\tau_M$ in this way across a range of gradients allows us to fit to (2.7) in order to extract the transverse spin diffusivity, $D_\perp$. Figure 4.5 shows that the magnetization decay time follows the predicted $(B')^{-2/3}$ scaling across a range of gradients.

Fitting to equation (2.7), we find $D_\perp = (1.08 \pm 0.09^{+0.17}_{-0.13})\hbar/m$, where the uncertainties are the statistical error from the fit and the systematic error from the gradient calibration respectively. The ability to tune the magnetic field gradient provides a direct measurement with a single fit parameter that does not rely on knowledge of cloud size, density, or trap frequency. This result is striking in that it is near the naive quantum limit of $\hbar/m$. However, an exploration of different initial temperature is required to determine if
this is truly limiting behaviour.

Still, this measurement of $D^\perp$ represents the lowest diffusivity measured in a Fermi gas in three dimensions. It is already below a measurement of longitudinal diffusivity [25] and still comparable even when compensating for inhomogeneous effects [15, 17, 7].

Interestingly, a recent measurement of transverse spin diffusion in two dimensions [20] shows a relaxation time roughly six times slower than what we observe at a comparable gradient and temperature. Using equation (2.7) this corresponds to a diffusivity $D_{2D}^\perp \sim 0.01\hbar/m$. This result highlights the interesting role of dimensionality on spin diffusion.

4.3 Dynamical and Inhomogeneous Effects

While this measurement is enabled by its dynamical nature, the evolution of the initial superposition presents additional problems that must be accounted for. One important aspect alluded to in Chapter 2 is the temperature dependence of the diffusivity. In these measurements, the relaxation of the magnetization is accompanied by a rise in temperature. This complicates the extraction of a trap wide diffusivity as the entire sample may not be thermalized. Assuming a diffusivity that depends on temperature implies that throughout the demagnetization the coefficient governing diffusion will be constantly changing. This makes measuring the temperature dependence, $D^\perp(T)$, difficult to isolate. Additionally, as the magnetization relaxes the polarization also changes. There is a prediction that $D^\perp$ changes with polarization [14]. This implies that the diffusivity further changes throughout the measurement and complicates making a trap wide measurement.
Chapter 5

Nonlinear Effects

In this section we explore the possibility to measure the Leggett-Rice effect in a three dimensional Fermi gas with a tuneable scattering length. The Leggett-Rice effect presents an interesting non-linear deviation from the simplest diffusion model for spin transport in a Fermi liquid. Since its derivation [22] this effect has been observed in the context of weakly interacting fermions in liquid helium [2, 5, 11] and ultracold atoms [13]. In these cases, the Leggett-Rice effect is manifest in spin-wave phenomena and anomalous coherence times. In this work, we present the first qualitative observation of the Leggett-Rice effect in a strongly interacting ultracold gas of fermions.

5.1 Leggett-Rice Effect for $M_z = 0$

The data presented in Chapter 4 falls into a unique category when discussing the Leggett-Rice effect. By construction, $M_z = 0$ forces solutions to follow the form (2.20). However, this data is also well fit to (2.8), which assumes $\mu = 0$. Together, this confuses the issue of whether a spin rotation effect should be present at unitarity. In fact, in the limit of a Boltzmann gas [24] the spin rotation parameter scales as $1/a_s$, where $a_s$ is the scattering length. This suggests that at unitarity, where the scattering length diverges, the spin rotation parameter should go to zero. Here, however, we work with a unitary Fermi gas.
Figure 5.1: The phases measured from Ramsey fringes for different gradients with $M_z = 0$ at unitarity. The data appears scattered around zero as expected from the form of equation (2.17).

in the degenerate regime where things might be very different.

As a preliminary test of our data analysis techniques we explore various methods to extract $\mu$ from the $\pi/2-\pi-\pi/2$ data at 202.1G using the parameters extracted from the Ramsey fringes. As seen in Figure 5.1, the phase obtained is scattered around zero over all of the gradients. As discussed, this is expected given the form of equation (2.17) and provides a noise floor for phase shifts the experiment can resolve. The data in Figure 5.1 also shows a transient effect manifest as an oscillation of the phase. This transient could be due to a number of systematic effects and will be discussed in detail at the end of this chapter.

Therefore, we attempt to fit the amplitude of the magnetization to equation (2.20) to extract $\mu$. This is shown in Figure 5.2a. Here $\mu$ is extracted from a fit to equation (2.20) over a range of gradients. The scatter in this data makes it very difficult to extract a value for $\mu$. While this data statistically excludes $\mu = 0$ at unitarity, there are a number of systematic issues that must be addressed and are discussed at the end of this chapter.
Figure 5.2: **a. µ versus gradient:** Values of µ extracted from fits of the amplitude of the magnetization to equation (2.20). µ = 0 is statistically excluded but a number of systematic effects impact this measurement. **b. Comparison of fits with (2.8) and (2.20):** The amplitude data is fit using equation (2.8) (blue) and (2.20) (red). When using the form for $M_z = 0$ the fit underestimates the amplitude effecting the curvature, µ, and the time constant extracted.

Figure 5.2b shows a comparison of a fit using (2.8) and (2.20). The fit using (2.20) causes an underestimation of the amplitude effecting the curvature, µ, and the time constant extracted.

## 5.2 Leggett-Rice Effect Versus Magnetic Field

As discussed, the Leggett-Rice effect may not be present at unitarity and if it is, it may be difficult to observe it in the amplitude of magnetization which is sensitive to a number of effects that could influence the extraction of µ. Therefore, as a “qualitative exercise”, we probe the relaxation of magnetization away from unitarity. For various magnetic fields from 201 G - 204 G we observe the relaxation of magnetization and using both the amplitude and phase, attempt to extract µ.

For each magnetic field value we apply a gradient of $\sim 35$ G/cm and carry out one of three pulse sequences, $\pi/4 - \pi - \pi/2$, $\pi/2 - \pi - \pi/2$, or $3\pi/4 - \pi - \pi/2$. We vary the pulse area of the initial pulse by changing the duration. The calibration performed optimized a
Figure 5.3: Amplitude and phase results from fitting the Ramsey fringes for data taken at 203 G. Data is taken with $\phi = \pi/4, \pi/2, 3\pi/4$ (blue, black, red). The solid lines represent equations (2.19) and (2.17) with a single hand picked value of $\mu$ and $\tau$ to optimize the overlap with the data. Qualitatively, the phase grows in opposite directions for $\phi = \pi/4$ and $\phi = 3\pi/4$ as expected from equation (2.17).

A 20\,\mu s $\pi$ pulse which correspondingly gives a $(\pi/4, \pi/2, 3\pi/4)$ pulse of $(5, 10, 15)\,\mu s$. When the initial pulse is $\pi/2$ we expect that the phase will remain scattered around $0^\circ$. However, in the case where the first pulse is $\pi/4$ or $3\pi/4$ we expect to observe cubic growth in the phase, the sign of which depends on the initial pulse and the sign of the scattering length.

Data at 203 G is shown in Figure 5.3. We see that as before, for $M_z = 0$ the phase does not grow cubically and is instead scattered around zero. More interestingly is the behaviour seen for different mixing angles. Qualitatively, the phase grows when $M_z \neq 0$ and the direction in which it grows depends on the sign of $M_z$ as predicted by equation (2.17). Hand selecting a single value of $\mu$ and substituting the amplitude measured at each hold time into equation (2.17) provides the solid lines in each of the plots for $\theta(t)$. The shaded region is optimized to best fit the data for $\theta(t)$ over a range of values for $\mu$. As can be seen in Figure 5.3, the data points lie well within this region for the short time data excluding the oscillatory behaviour present in the first few data points. To produce the solid lines in the plots of the amplitude, a single value for $\tau$ is also handpicked to best fit the data when substituted into equation (2.19). This provides a single value of $\tau$.
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Figure 5.4: Amplitude and phase results from fitting the Ramsey fringes for data taken at 201 G. Data is taken with $\phi = \pi/2, 3\pi/4$ (black, red). The solid lines represent equations (2.19) and (2.17) with a single hand picked value of $\mu$ and $\tau$ to optimize the overlap with the data.

and $\mu$ at each field value that fits all of the data.

We do not perform a fit to extract $\mu$ for a few reasons. First, extracting $\mu$ from the amplitude has shown to be difficult as the amplitude is sensitive to many dynamical effects. Further, an underestimation of the initial amplitude with which the curves are normalized can cause the values predicted for $\theta(t)$ to be significantly changed. The amplitude is also systematically below the theoretical value for each pulse area (although this is more apparent in the $\pi/4$ and $3\pi/4$ cases). In addition, a clear transient effect can be seen in the phase data for short hold times. This apparent oscillation limits the statistical certainty with which a fit can be performed.

This behaviour is reproducible over a range of magnetic field values. In particular, when tuning the field to the repulsive side of resonance, the direction in which the phase grows changes as the scattering length has changed signs from negative to positive. This is shown in Figure 5.4. Again, $\mu$ and $\tau$ are handpicked to best optimize the overlap with the data points.

Having successfully observed the qualitative nature of $\mu$ across the Feshbach resonance we attempt to quantify this behaviour versus magnetic field. Repeating the measurements described above for more field values we can begin to resolve the dependence of $\mu$ on
Chapter 5. Nonlinear Effects

Figure 5.5: The handpicked values of $\mu$ and $\tau$ extracted from Ramsey data at various fields and for various initial mixing angles. The gray data point represents data taken with r.f. pulses of the same pulse area but lower amplitude and a gradient of $\sim 15$ G/cm. magnetic field. This is shown in Figure 5.5.

We plot the handpicked values for $\mu$ and $\tau$ versus magnetic field and can, despite the qualitative treatment of the data, begin to see interesting features. In the data for $\tau$, a local maximum appears to be present at unitarity. Using equation (2.7) this would correspond to a local minimum in the diffusion constant, $D^\perp$. Surprisingly, this matches the qualitative behaviour recently observed in two dimensions [20] despite the drastic differences in the value for the diffusivity at unitarity.

The data for $\mu$ also shows some striking features. In particular, it appears that $\mu$ will intersect zero for some field. In the Boltzmann limit the zero crossing should occur at unitarity. While more measurements are needed before a quantitative conclusion is made, it seems likely that $\mu \to 0$ for some value of the scattering length. In addition, the behaviour away from resonance is interesting. Assuming a Boltzmann gas where $\mu \propto 1/a_s$ we expect the spin rotation parameter to diverge at the zero crossing of the scattering length (in this case 209G). However, the data collected qualitatively supports a saturation of the spin rotation parameter as it tends to the zero crossing. This difference is not unexpected as the Boltzmann limit does not hold for a strongly interacting degenerate Fermi gas.
5.3 Current Challenges and Limitations

While the data presented shows that we have observed the Leggett-Rice effect, there are still a number of systematic improvements that could enhance the quality of our data. An important issue alluded to is a transient effect in the measurement of the phase of the Ramsey fringe.

This transient could be caused by pickup on one of the nearby chip wires causing an oscillation in the magnetic field. This could translate to an offset in the relative phase of the atoms with respect to the source. A possible solution to this problem is to include an r.f. choke on one of the chip wires. This would hopefully suppress any unwanted pickup. The transient could also be minimized by decreasing the amplitude of the r.f. pulses while increasing the duration (to keep the pulse area the same). The result for \( \mu \) from a data set taken in this way is shown as the gray point in Figure 5.5. The quality of this particular data set was already improved over the other data sets presented here.

Another issue that needs to be addressed before a quantitative measurement can be performed is the r.f. pulse calibration. The amplitude data presented shows a significant deviation from the ideal initial amplitude for each of the first r.f. pulses. This is particularly apparent in the data for \( \phi = \pi/4 \) and \( \phi = 3\pi/4 \). In these cases the amplitude should be the same, \( A(0) \approx 0.85 \). However, the data shows a larger amplitude for a \( \pi/4 \) pulse and a smaller amplitude for a \( 3\pi/4 \) pulse.

The deviation of the initial amplitude from the theoretical value is most likely due to an error in the echo pulse. While the pulse area has been well calibrated, a transient caused by the first r.f. pulse or a timing error of the echo pulse could impact the measured phase and amplitude. Possible timing issues have been excluded implying that the most likely effect is again a transient in the field. Possible solutions for this transient effect have been described.
Chapter 6

Conclusions

This report has presented novel explorations of the spin transport in a strongly interacting Fermi gas. Namely, the experiments presented have shown the capability to measure the transverse spin diffusivity via a spin-echo experiment reminiscent of the pioneering work done in liquid helium. Further, a result $D^\perp \approx \bar{h}/m$ at the naive quantum limit suggests that unitary scattering could suppress transport with a universal lower bound in three dimensions.

Additionally, we have shown that nonlinear effects beyond the simplest diffusion model can be measured using the same spin-echo techniques. This presents the first observation of the Leggett-Rice effect in a strongly interacting ultracold degenerate Fermi gas. While these results remain relatively qualitative in their current form, they strongly suggest that a measurement of the spin rotation parameter, $\mu$, could be performed for a range of scattering lengths as well as for various polarizations.

This work suggests a few directions of research to pursue. In order to demonstrate the universality of the diffusion measurement presented in Chapter 4 this experiment should be completed over a range of temperatures and polarizations. This would allow for direct comparison to theory [14] as well as provide a clue towards whether spin diffusion is suppressed to a universal lower bound by unitary interactions.
In addition, a direct measurement of the spin rotation parameter is possible and could be pursued after addressing the limitations described in Chapter 5. It would be interesting to study the behaviour of the spin rotation parameter for various scattering lengths for a degenerate Fermi gas. A concurrent measurement of the spin rotation parameter versus scattering length at a higher temperature would also provide a comparison to the theoretical predictions for a Boltzmann gas [24].
Bibliography


