Testing of a Microscope for Site-Resolved Imaging of Atoms in an Optical Lattice

by

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A thesis submitted in conformity with the requirements for the degree of Masters of Science
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Chapter 1

Introduction

This thesis describes the testing of two high numerical aperture objective lenses targeted at the imaging of individual $^{40}\text{K}$ atoms in an optical lattice with a period of about 527 nm. The results help complete one main aspect of the construction and testing of a microscope stack currently being built by the Ultracold atoms group at the University of Toronto.

1.1 Hubbard Model and Optical Lattices

One of the simplest models that describes the quantum motion of electrons in a crystalline solid is the Hubbard model [1]. It takes into account the ability for the electrons to hop between nearest-neighbour atoms as well as the repulsive Coulomb interaction between two electrons in a single atomic orbital. The Hubbard model Hamiltonian for fermionic particles is given by

$$H = -t \sum_{\langle i,j \rangle, \sigma} \left( c_{i\sigma}^\dagger c_{j\sigma} + c_{j\sigma}^\dagger c_{i\sigma} \right) + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

where $\langle i,j \rangle$ represents the nearest-neighbour lattice sites, $\sigma$ is the spin, $c_{i\sigma}^\dagger$ is the annihilation (creation) operator for a particle with spin $\sigma$ on site $i$, and $n_{i\sigma} = c_{i\sigma}^\dagger c_{i\sigma}$ is the number operator. The model parameters, $t$ and $U$, represent the tunneling strength between adjacent sites and the on-site interaction strength, respectively.
Depending on the chemical potential, the temperature, and the relative interaction strength $U/t$, the Hubbard model predicts metallic or insulating phases. One of these phases is the Mott insulator, which is predicted for a mean filling of $\langle n_{\sigma i} \rangle = \frac{1}{2}$ and for large interaction strength $U/t \gg 1$. The repulsion of the particles leads to a many-body state with exactly one particle per site. An outstanding question is whether doping of this state away from half-filling leads to superconductivity.

An ideal system to tackle such open questions in a direct quantum simulation is an ultracold quantum gas in an optical lattice [2–4].

An optical lattice is a sinusoidal potential created by interfering coherent laser beams. The Hubbard model is a good description of ultracold fermions in an optical lattice; therefore, they provide a way to experimentally simulate its physics. While the electrons in a solid feel the Coulomb potential created by ions, fermions in an optical lattice feel a periodic potential formed by interfering laser beams through the optical dipole force. Understanding the movement of and interactions between the atoms in the lattice leads to an understanding of the physics of the Hubbard model.

One of the main benefits of an optical lattice is the ability to easily control the parameters in the Hubbard model by adjusting the lattice depth. Other methods of testing condensed-matter theories, such as probing fabricated solid crystals, are not as easily realized, and computer simulations are not possible for a large number of atoms.

The standard method for imaging atoms in an optical lattice is through time-of-flight absorption imaging, which probes the ensemble-averaged momentum distribution of the atoms. With single-site resolved in-situ imaging of ultracold fermionic atoms in an optical lattice, correlations between lattice sites could be directly measured, leading to an unprecedented probe of the physics in and beyond the Hubbard model. This approach has already successfully been demonstrated with bosonic $^{87}$Rb [5–7].
1.2 Overview of Thesis

This thesis is structured as follows. Chapter 2 discusses the experimental design in the context of high-resolution imaging of the atoms in the lattice. Chapter 3 gives an overview of the Fourier theory of imaging and the necessary background physics for understanding the rest of the thesis. Chapter 4 and 5 describe the two tests performed with the microscope to determine the resolution and quality of imaging. Chapter 6 concludes with a summary of the results and suggestions for further testing.
Chapter 2

Experimental Context

The Ultracold atoms group at the University of Toronto is currently striving toward creating an optical lattice of fermionic potassium-40 (\(^{40}\text{K}\)) atoms. The work described in this thesis was performed in parallel with the group’s efforts on building the experimental apparatus for cooling and trapping of \(^{40}\text{K}\) in hopes of having a completed microscope ready for imaging by the time the atoms are first loaded into the lattice.

A schematic diagram of the lattice chamber is shown in Fig. 2.1. Three mutually orthogonal, retro-reflected laser beams with wavelength of approximately 1054 nm interfere to create a three-dimensional cubic lattice with a spacing of 527 nm. Additional laser beams, sent through the lattice in the horizontal plane perpendicular to the imaging axis, excite the atoms, causing them to fluoresce at 405 nm, among other wavelengths. The polarization and wavelength of this fluorescence beam are chosen to laser-cool the atoms during the imaging [8]. The fluorescent light is captured by a high numerical aperture objective lens which is positioned about a millimeter above a 0.2 mm thick sapphire imaging window.

The level diagram for \(4S_{1/2} \rightarrow 5P_{3/2}\) transition used in imaging \(^{40}\text{K}\) is shown in Fig. 2.2. As resolution is proportional to wavelength, in order to resolve single sites in the 527 nm lattice, we intend to image the atoms by capturing 405 nm fluorescence, the smallest wavelength available with \(5P\) excitation. The conventionally used cooling transition at 767 nm has a 30
times higher photon scattering rate but a resolution limit that is almost a factor of two larger when compared to the 405 nm transition. Due to the increased number of 767 nm photons per atom, we may initially align our microscope using red fluorescence and then switch to blue fluorescence to resolve single sites. For this reason, this thesis discusses resolution tests performed with both 405 nm and 767 nm light.
2.1 Microscope Stack Design

The microscope optics must be sensitive to the weak fluorescent light emitted by the atoms and must be diffraction-limited with a large numerical aperture in order to achieve a small enough resolution to distinguish adjacent lattice sites. The optical lattice is contained in a vacuum system and the objective lens is supported outside; the two are separated with a 0.2 mm sapphire window. The physical constraints require a long working distance (the distance from the front of the objective to it’s focus), and the small desirable resolution necessitates a large numerical aperture.

The Ultracold atoms lab has purchased two objective lenses that are possibly suitable for the imaging of $^{40}\text{K}$ in an optical lattice. An off-the-self Zeiss objective lens (441370-9970) was originally purchased in 2007. The Zeiss objective has previously been tested by a former student and was found to not be diffraction limited with blue light [10]. As a result of these tests, an objective, with a long working distance and large numerical aperture (NA), was custom made.
for this experiment by Special Optics in 2010 and it is designed to be diffraction limited for 405 nm light. The working distance is 2.43 mm, the NA is 0.6, the magnification is $64 \times$, and the lens is corrected to image through the 0.2 mm sapphire vacuum window. We wish to not only have a thorough understanding of how different imaging conditions affect the resolution of the Special Optics objective, but also to compare its performance with the Zeiss objective.

The Zeiss objective lens was previously found to be almost diffraction limited for red illumination, but not for blue. It has a working distance of 2.2 mm, a NA of 0.75, and a magnification of $63 \times$. Both the Zeiss objective and the Special Optics objective are infinity collimated and have a specific tube lens designed to focus down the light onto the imaging plane.

The rest of the microscope stack principle is shown in Fig. 2.3, which shows a photograph of the constructed stack on top of a spare vacuum flange. The stack, which controls the positioning of the objective lens, consists of three-axis manual translation stage (Newport M406), two-axis tilt stage (Newport TTN80) and a z-piezo stage (PI P-733.Z) to control focus. No tests have been made with the stack fully constructed; instead, we used a more flexible testing setup affixed to the optical table. In the current design, the tube lens is positioned 30 cm away from the back of the objective lens.
Figure 2.3: Photo of constructed microscope stack on a spare vacuum flange showing translation stages which act to control and stabilize the position of the objective lens.
Chapter 3

Fourier Theory of Imaging

3.1 Fraunhofer Diffraction by a Circular Aperture

The performance of an imaging system is subject to lens imperfections, misalignment, and diffraction. Aberrations caused by imperfections and misalignment can be reduced to a negligible amount by careful design; however, diffraction is a result of the wave nature of light, and therefore sets a fundamental limit. The response of an imaging system to a point source, analogous to an impulse response function in electronics, gives a measure of the resolution ability of the imaging system. This response, which will have a finite width due to diffraction, is referred to as the point spread function (PSF).

In the Fraunhofer limit, the amplitude of the diffraction pattern for an arbitrary aperture is given by [11, 12],

\[ \psi(u, v) = \int \int f(x, y) \exp[ik(xu + yv)] \, dx \, dy, \] (3.1)

which is just the two-dimensional Fourier transform of the mask transmission function, \( f(x, y) \), defined to be zero where the mask is opaque and unity where it is transparent. To determine the PSF, we solve Eq. 3.1 for a circular aperture, corresponding to the limiting aperture of the microscope. The resulting focal plane intensity distribution is known as the Airy disk, given
by

\[ I(\theta) = I_0 \left( \frac{2J_1(ka \sin \theta)}{ka \sin \theta} \right)^2, \]  

(3.2)

where \( \theta \) is the angle of observation, \( I_0 \) is the maximum intensity, \( J_1 \) is the Bessel function of the first kind, \( k \) is the wavenumber, and \( a \) the radius of the diffracting aperture. Figure 3.1 shows the Airy pattern in one-dimension as a function of \( ka \sin \theta \). The first minimum occurs at \( ka \sin \theta \approx 3.8317 \).

Figure 3.1: Diffraction by a circular aperture results in an Airy pattern, defined in Eq. 3.2.

The standard definition of resolution used in optics is the Rayleigh criterion. It states that two incoherent point sources, which each produce an Airy pattern, are to be called “resolved” when the location of the first minimum of one Airy disk corresponds to the maximum of the other, as shown in Fig. 3.2. For a diffracted limited system, the Rayleigh resolution is defined as

\[ R = \frac{1.22 \lambda}{2NA}, \]  

(3.3)

where NA is the numerical aperture of the imaging system. Resolution can be reduced by decreasing the wavelength of light or increasing the microscope NA.
3.1.1 Optical Transfer Function

In the same way that the PSF is analogous to the impulse response function in electronics, the optical transfer function (OTF) \([13, 14]\) is analogous to the frequency response function. The OTF is the Fourier transform of the point spread function,

\[
\text{OTF}(u,v) = \int \int \text{PSF} \cdot \exp[-ik(xu+yv)] \, dx \, dy.
\]

(3.4)

and can be separated into amplitude modulation and phase components, \(\text{OTF} = \text{MTF}e^{i\text{PTF}}\). The modulation transfer function (MTF) is often used to specify the resolving abilities of an imaging system, and thus I have included this short discussion here although I do not make any measurements of the MTF. The MTF is given by the ratio of the image contrast and the object contrast, where contrast is defined by

\[
C = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}},
\]

(3.5)

and \(I_{\text{max}}\) and \(I_{\text{min}}\) corresponds to the maximum and minimum intensities. The MTF describes the magnitude response of an imaging system to a sinusoidal object of different frequencies. For a diffracted limited system with a circular aperture, the MTF is given by

\[
\text{MTF}(\Omega) = \frac{2}{\pi} \left[ \cos^{-1} \Omega - \Omega \sqrt{1 - \Omega^2} \right],
\]

(3.6)
where \( \Omega \) is the normalized spatial frequency, \( \Omega = \nu / \nu_{\text{cutoff}} \) with \( \nu_{\text{cutoff}} = 2NA / \lambda \). Figure 3.3 shows the MTF as a function of \( \nu / \nu_{\text{cutoff}} \) for a diffracted limited system. The Rayleigh resolution corresponds to a frequency of \( \nu / \nu_{\text{cutoff}} = 1/1.22 \), which gives a modulation value of almost 9\%, as shown in Fig. 3.3.

![Figure 3.3: Modulation transfer function for a diffracted limited system. The Rayleigh resolution corresponds to a normalized frequency of \( \nu / \nu_{\text{cutoff}} = 1/1.22 \) and is shown by the dashed gray line to correspond to an MTF of almost 9\%.

The phase transfer function (PTF), which is zero for a diffraction limited system, describes the change in phase between the object and the image and thus relates to asymmetric aberrations in the PSF.

### 3.1.2 Three-Dimensional Diffraction Pattern

The two-dimensional diffraction pattern of a circular aperture at the focal plane of a lens is given by The Airy disk, as described in the previous section. The three-dimensions diffraction pattern from a circular aperture is given by [15]

\[
I(p, q) = \left( \frac{2I_0}{p} \right)^2 \left\{ \sum_{s=0}^{\infty} (-1)^s \left( \frac{p}{q} \right)^{2s+1} J_{2s+1}(q) \right\}^2 + \left[ \sum_{s=0}^{\infty} (-1)^s \left( \frac{p}{q} \right)^{2s+2} J_{2s+2}(q) \right]^2 \tag{3.7}
\]

where \( I_0 \) is the peak intensity at the focus, and \( p \) and \( q \) are dimensionless variables

\[
p = \frac{2\pi}{\lambda} NA^2 z, \quad q = \frac{2\pi}{\lambda} NA \sqrt{x^2 + y^2}. \tag{3.8}
\]
Figure 3.4: Equi-intensity lines showing the $xz$-distribution of the three-dimensional diffraction pattern by a circular aperture. The Airy disk corresponds to the pattern observed in the focal plane. [15]

Figure 3.4, from Born and Wolf [15], shows a cross-section of this pattern, where the two dimensional Airy disk is located at the focal plane, $z = 0$.

The depth of focus is a measure of the axial distance over which an object is considered to be in focus. It is defined as half of the axial distance to the first minimum of intensity to the left or the right the focal plane. The first minimum of Eq. 3.7 is located at a distance of $z_{\text{min}} = \frac{2\lambda}{NA^2}$, and thus the depth of focus $D$ is given by

$$D = \frac{\lambda}{NA^2}. \quad (3.9)$$

3.1.3 Aberrations

Aberrations refer to the deviation of the performance of an optical imaging system compared to that expected from Gaussian optics [16]. Aberrations can be either chromatic or monochromatic. The three primary monochromatic aberrations which deteriorate an image are spherical aberrations, coma and astigmatism.
Spherical aberrations, which occur for objects along the optical axis of the imaging system, are a result of the dependence of the focal length of a lens on the distance to the optical axis (see Fig. 3.5). When imaging a point source, this effect shifts light out of the central peak of the Airy disk and into the surrounding rings. Spherical aberrations can be reduced by lowering the numerical aperture of an imaging system with the addition of diaphragms, often, however, at the cost of a worse resolution.

Figure 3.5: Ray diagram showing spherical aberrations. [15]

Coma occurs for off-axis imaging as a result of rays passing through a lens at different distances from the optical axis being focused to different spots on the imaging plane. Coma causes the point spread to appear as if it was imperfectly illuminated, with the Airy rings appearing darker on one side of the image. Figure 3.6 shows the result of coma in the imaging plane. The same techniques used to reduce spherical aberrations often work for coma as well.

Astigmatism, another off-axis imaging effect, can be understood by introducing the sagittal and tangential planes. The tangential plane contains the object and the optical axis, and the sagittal plane is perpendicular to it, containing the object and intersects the optical axis at the entrance pupil (see Fig. 3.7). The result of the difference in focus for these two planes leads to a blurred image in between the two foci, or an elliptical PSF when offset from the center of the foci.
3.1.4 Diffraction from an Array of Circular Apertures

An array of apertures can be mathematically represented by the convolution of an array of delta-functions with one aperture. The convolution theorem states that the Fourier transform of the convolution of two functions is equal to the product of the two functions’ Fourier transforms. Thus, the diffraction pattern for an array of circular point-like apertures is given by the product of the PSF and the Fourier transform of an array of delta-functions,

\[ \psi(u) = \text{PSF} \cdot \sum_{m=-\infty}^{\infty} \delta(u - 2\pi m/d), \]  

(3.10)

where \( m \) is referred to as the order of diffraction, and \( d \) is the array spacing. The resulting diffraction pattern has peaks of maximum intensity located at \( m\lambda = d(\sin \theta - \sin \theta_0) \), where \( \theta_0 \) is the angle of incident light.

To reconstruct an image of a diffraction grating, at least two of the diffracted orders must be collected by the lens; however, the spatial frequency information contained in the uncollected higher order peaks is lost. To achieve better resolution, one can illuminate with a large number of different angles of incidence, which acts to wash out the diffraction pattern and to reduce the loss of information. Köhler illumination, as will be described in Sec. 5.2.1, is designed to do exactly this.
3.1.5 Image Formation

For incoherent light, an extended transmission pattern can be considered a collection of points, each of which produces an independent PSF in the focal plane of the imaging lens. Mathematically, this is represented by the convolution of the spatial transmission pattern of the object and the point spread function.
One goal of this thesis was to compare the performance of the Special Optics and the Zeiss objective lenses. We aimed to determine the imaging ability of each objective at both wavelengths of 405 nm and 767 nm. This was done by imaging a pinhole with a diameter of 0.5 µm in transmission, illuminated with a laser source of either 405 nm or 767 nm. The pinhole was ordered from National Aperture. They quote the hole diameter and tolerance as $0.5^{+0.3}_{-0}$ µm.

For red illumination, the pinhole size is smaller than the diffraction limited resolution of both objective lenses, and thus the pinhole can be thought of as an approximate point source. The diffraction limited resolution for both of the two objective lenses at 405 nm light is less than 500 nm, and thus the hole should be resolvable and no longer approximates a point source. One then expects to image the convolution between the pinhole and the point spread function, as described in Ch. 3.
4.1 Experimental Setup and Alignment

The experimental setup for the pinhole measurements is shown in Fig. 4.1. The pinhole is placed at the focus of the objective lens, and when illuminated from behind with a laser beam, the small aperture nearly acts as a point source of incoherent light. Both objectives are infinity focused, and thus the distance between the tube lens and the objective does not need to be very accurate and is varied throughout our tests (see Sec. 5.3.5). The tube lens focuses the collimated light down onto a MicroPix M640 charge coupled device (CCD) camera to give a small light spot which, in the case of red illumination, nearly corresponds to the PSF of the imaging system.

Figure 4.1: Experimental setup for measuring the illumination from a 0.5 µm pinhole. Shown are the pinhole mounted on an xyz-translation stage, the microscope assembly with the Special Optics objective, an iris diaphragm, the tube lens, and the CCD camera.

Even if the CCD is not positioned one focal length away from the tube lens, a focused image can be formed by adjusting the position of the objective with respect to the pinhole. However, the magnification of the microscope will be affected, and the microscope will not be infinity collimated. Therefore, the distance between the tube lens and the CCD chip was set by focusing down a collimated laser beam and minimizing the spot size for each combination of wavelength and objective.

As shown in Fig. 4.1, the objective, tube lens and CCD were axially supported using a cage system, which was aligned to be parallel to the table with the help of a laser beam. The pinhole
was then positioned to be perpendicular to the optical axis of the microscope by aligning the reflection off of the back surface to overlap with the incident laser beam. Pedestal mounts were used to increase the stability of the microscope. All of the flexibility in aligning the pinhole comes from it being mounted to a three-axis translation stage, as shown on the left side of Fig. 4.1. The waist of the illuminating laser beam was large enough so that the pinhole could be moved through the entire field of view and could maintain the same transmitted intensity.

The objectives were mounted in a Thorlabs SM1Z translation stage, which has an actuator knob with $1 \mu m$ increments, and this served as the focusing element of the microscope. An iris, which allowed us to limit the NA of the microscope, was located behind the objective lens in the infinity focused region.

Originally, we had difficulty in finding the pinhole with our microscope. If the pinhole is displaced from the focal plane of the microscope by more than a few tens of microns, the blurred diffraction pattern is extremely weak and only visible when the CCD camera’s exposure time and gain are set to maximum values. To help with the alignment of the 0.5 $\mu m$ pinhole, we first image a 25 $\mu m$ pinhole. This allows us to know the approximate location of the microscope’s optical axis and focal plane. After substituting the 0.5 $\mu m$ pinhole into the same mount, and returning to the same focal plane, a signal can be found by slowly scanning the close vicinity with the $x$ and $y$ translation stages.

### 4.1.1 Measuring Magnification

When extracting a measure of resolution from a CCD image of the pinhole, it is essential to know the magnification of the microscope. As mentioned in Sec. 4.1, a change in the distance between the CCD and the tube lens can alter the magnification of the microscope, and as such, one should measure the magnification for a given microscope setup.

For each objective lens, the optimal distance from the tube lens to the CCD was found by focusing a well-collimated beam of light onto the CCD and moving the tube lens until the spot size was of minimum size. The magnification of the microscope was then measured for each
wavelength. Table 4.1 shows the optimal distance from tube lens to CCD and the resulting magnification for each objective. The quoted magnification for the Special Optics objective is $64 \times$, and $63 \times$ for the Zeiss objective. The distance is measured from the front face of the CCD camera to the front of the mount to which the tube lens is attached (length $L_1$ in Fig. 4.1).

<table>
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<tr>
<th>Objective and Wavelength</th>
<th>$L_1$(cm)</th>
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<tr>
<td>Zeiss and 405 nm</td>
<td>16.15</td>
<td>61±3</td>
</tr>
<tr>
<td>Zeiss and 767 nm</td>
<td>16.15</td>
<td>65±3</td>
</tr>
<tr>
<td>Special Optics and 405 nm</td>
<td>14.30$^1$</td>
<td>65±3</td>
</tr>
<tr>
<td>Special Optics and 767 nm</td>
<td>19.60</td>
<td>65±3</td>
</tr>
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Table 4.1: Measured magnifications for optimal tube lens-to-CCD distance as set with 405 nm light.

We measure magnification by translating the object a known amount, and measuring the displacement of its image. The pinhole was translated laterally by a precise distance of $60 \pm 3 \, \mu m$ using a Newport UMR3.5 translation stage with a BM11.5 micrometer screw. We obtain an estimate for the statistical error by repeating the measurement five times and taking the standard deviation.

Unfortunately, the importance of carefully setting the distance between the tube lens and CCD camera was realized after the majority of the point spread function data was taken. As there was not sufficient time to repeat all measurements, only the evaluations of the best-effort resolution were performed with the distances and hence magnifications listed in Table 4.1. The other tests are of a more qualitative nature and an overall scaling of the width due to a change in magnification will not affect the important conclusions here. However, one should be cautious, as an imperfect distance between the tube lens and CCD could lead to enhanced

$^1$For the Special Optics lens and 405 nm light, we used the quoted focal distance of the lens to set the distance to the CCD instead of the distance found with a collimated laser beam. This was in a last minute attempt to correct spherical aberrations seen within the microscope, which might have resulted from setting the distance using a slightly diverging laser beam.
monochromatic aberrations.

### 4.2 Resolution Measurements

Assuming completely incoherent illumination, the diffraction pattern of the pinhole is the convolution of the Airy disk and a 0.5 \( \mu \text{m} \) diameter hole (see Sec. 3.1.5). Figure. 4.2 shows the resulting convolution for a diffraction limited Airy disk in one-dimension, for each objective and wavelength combination. This model may be an oversimplification, as the transmission of light through the pinhole may be more complicated due to the effect of surface plasmons [17] and the coherence of the laser light used for illumination.

For each objective and wavelength combination, three different measurements were performed to determine the quality of the microscope: determining the resolution from a focused image, the resolution as a function of objective numerical aperture, and a qualitative measurement of aberrations.

For all of these tests, the CCD settings remained constant. The exposure time was set to the camera’s minimum of 33 ms and the read-out gain was set to zero. The intensity of the illumination beam was adjusted to achieve a well-illuminated pinhole without saturating the CCD.

#### 4.2.1 Algorithm to Determine Resolution

To determine the resolution, the PSF was centered in the camera’s field of view in the attempt to minimize aberrations from coma and astigmatism, and the objective was axially translated in micrometer steps through the focus. Multiple images were taken at each objective position to account for instabilities in the system. The images around the focus were fit with the two-dimensional Airy disk defined by

\[
I = I_0 \left( \frac{2J_1(\rho)}{\rho} \right)^2 + c, \quad (4.1)
\]
Figure 4.2: Comparison of bare and convoluted Airy disks. The blue curve shows the convolution of one-dimensional diffracted limited Airy disk (light blue) and a 500 nm hole (gray) for each wavelength and objective. 

- **a.** Special Optics at 405 nm.
- **b.** Special Optics at 767 nm.
- **c.** Zeiss at 405 nm.
- **d.** Zeiss at 767 nm.

The first minimums of the convolution are found at 632 nm, 943 nm, 565 nm and 860 nm for **a.**, **b.**, **c.**, and **d.**, respectively. The numerical aperture of the Special Optics objective is 0.6, and is 0.75 for the Zeiss objective. Due to the smaller wavelength, the blue diffraction pattern is altered more by the convolution than the diffraction pattern from red light.

\[
\rho = \sqrt{\left(\frac{x-x_0}{w_x}\right)^2 + \left(\frac{y-y_0}{w_y}\right)^2}.
\]  

(4.2)

The six fit parameters are peak intensity \(I_0\), the constant offset \(c\), the center coordinates \(x_0\) and \(y_0\), and the widths \(w_x\) and \(w_y\). The widths defined here can be converted to the distance of the first minimum by multiplying by 3.8317. The fitting algorithm allows for an asymmetric Airy disk, with different widths in the \(x\) and \(y\) directions.

As our CCD images are actually a convolution of the pinhole and the true PSF, we need to convert the fit width into an unconvolved Airy width to get a measure of the resolution of
the system. To determine the conversion factors, we carry out fits of Airy disks to a pinhole pattern convoluted with Airy disks of different widths in one-dimension. Figure 4.3 shows the resulting conversion curve, where the x-axis is the width of the fitted pattern, normalized by the pinhole radius, and the y-axis is the corresponding ratio of the original Airy width to the fitted width.

![Figure 4.3: Curve representing the conversion between the width of the fitted Airy function and the corresponding unconvoluted PSF width.][1]

Ideally, we would define the Airy fit which gives the lowest reduced chi-squared value to be the focused PSF. However, when the PSF has strong aberrations, lowest reduced chi-squared value does not always correspond to the focused PSF. We therefore also look at which fit shows the lowest width and maximum intensity. The resolution is taken as the radial distance to the first minimum of the unconvolved Airy disk, as stated by the Rayleigh criterion (Sec. 3.1).

The main error in the resolution stems from the error in the magnification. To convert from a width measured in pixels to a real space width, one multiplies by the pixel size and divides by the magnification. Therefore, the typical 5% error in magnification (see Table. 4.1) corresponds to about a 5% error in resolution measurement. The other uncertainties associated with these measurements are in the size of the pinhole, the sampling of the imaging plane, and the statistical error in the fit.

Our assumption of the hole diameter being 0.5 µm, gives us a worst-case estimate of the
resolution as the quote tolerance on the diameter is \(0.5^{+0.3}_{-0.0} \mu m\). If the hole is larger than 0.5 \(\mu m\), then the unconvoluted PSF would have to be more narrow to create the same fitted Airy width.

As images are only taken at objective axial distances that differ by 1 \(\mu m\), we most likely do not take an image at the actual focus of the microscope, and thus the resolution we obtain is an overestimate.

As the systematic errors in these measurements are quite large, we ignore the statistical error associated with the fits. As the magnification uncertainty is the only error which we can confidently quantify, we report the error in the resolution solely as the error in magnification. However, as mentioned above, the uncertainty in the hole size and image plane sampling allow us to understand our resolution measurements as an upper limit.

4.2.2 Minimum Resolution from a Focused Pinhole Image

As discussed in Sec. 3.1.2, the diffraction pattern no longer resembles an Airy pattern as the pinhole moves away from the focal plane. Figure 4.4 shows eight diffraction patterns taken with the Special Optics objective and blue illumination. The objective position along the microscope axis is increased by 1 \(\mu m\) for each image. The asymmetry of the diffraction pattern around the focal plane indicates spherical aberrations.

We fit an Airy disk to each of these diffraction patterns, and ideally, we would determine the focus by the fit which gives the lowest reduced chi-squared. Figure 4.5a shows the fitted Airy width and reduced chi-squared as a function of the objective focus for Special Optics at 405 nm. Unfortunately, for this data set, the minimum in reduced chi-squared corresponds to a diffraction pattern as seen in Fig. 4.4b, and is not the focus. We therefore determine the focus by finding the minimum fitted width and check to make sure it corresponds to an image that has maximum intensity. In Fig. 4.5a., the values for the fitted width decrease steadily as the objective displacement is increased, and the fitting algorithm fails for the diffraction pattern located at the next axial position; therefore, we determine the diffraction pattern corresponding
Figure 4.4: Diffraction pattern as a function of objective’s optical axis displacement for the Special Optics objective illuminated with 405 nm light. The objective is translated by $1 \, \mu\text{m}$ in between each picture. The asymmetry around the focused image, determined to be e, is an indication of spherical aberrations.
the axial position of 2 µm to be in focus.

![Graph](image)

**Figure 4.5:** **a.** Special Optics with 405 nm illumination. **b.** Special Optics with 767 nm illumination. The blue curve shows the fitted resolution as a function of the objective focus and the green curve shows the related reduced chi-squared. We would expect these two curves to reach a minimum at the focus of the objective.

It should be noted that the values plotted along the x-axis for objective displacement are arbitrary, and the zero crossing has no special significance. Only the relative positions are important. Also, all resolution values displayed in plots correspond to the width of the fitted Airy function, without yet applying the conversion into a proper measure of resolution.

The PSF, which corresponds to the focused diffraction pattern for Special Optics at 405 nm, is shown in Fig. 4.4e. The Airy disk fit gives us a width of 650±30 nm in the x-direction and 680±30 nm in the y-direction. We use the conversion curve in Fig. 4.3 to convert these widths into a resolution of 490±40 nm and 530±40 nm in the x and y-direction, respectively. The x and y-directions correspond to the coordinates of the CCD pixel array and do not coincide with possible symmetry axes within the pinhole image. Although this diffraction pattern has minimum width and maximum intensity, when compared to images taken at other objective axial positions, the reduced chi-squared is a factor of two larger than the other fits. We attribute this to the fact that the diffraction pattern is highly distorted due to spherical aberrations.

The results for the resolution and reduced chi-squared for the Special Optics objective and 767 nm light is shown in Fig. 4.5b. The local minimum of both curves at an axial displace-
ment of 1 µm corresponds to the focus of the point spread function, as verified by eye, and the obtained resolution, given as an average of the x and y-direction, is 620±50 nm. The asymmetry of the PSF, found from the ratio of widths along the two axes, is 2% for this fit. Figure 4.6 shows the diffraction pattern as a function of objective optical axis displacement for red illumination and the Special Optics objective.

![Image](image_url)

Figure 4.6: Diffraction pattern as a function of objective’s optical axis displacement for the Special Optics objective illuminated with 767 nm light, as in Fig. 4.4. The focus was determined to correspond to the image shown in c.

Figures 4.7a and b show the fitted width and reduced chi-squared data for the Zeiss objective lens for 405 nm and 767 nm illumination, respectively. Figure 4.7a shows a minimum
in resolution at the same location of a maximum in reduced chi-squared, similar to what we saw in Fig. 4.5. Again, the intensity was a maximum for this pattern, and, in the same way as with the blue illumination of the Special Optics objective, we attribute this large reduced chi-squared value to the fact that the diffraction pattern had large aberrations. The resolution obtained from the fit is $350 \pm 60$ nm, with an asymmetry in the $x$ and $y$-direction of almost 20%.

The diffraction pattern as a function of objective axial displacement is shown in Fig. 4.8.

Figure 4.7: a. Zeiss objective illuminated with 405 nm. b. Zeiss objective illuminated with 767 nm. The minimum in resolution gives what we deem to be focused.

The results for red illumination with the Zeiss objective are shown in Fig. 4.7b. This was the only combination of objective and wavelength in which we observed a somewhat symmetric diffraction pattern as the objective’s focus scanned through the pinhole. The widths were evaluated for the axial displacement position of 0, which again corresponded to a maximum in intensity and a minimum in width, and the obtained resolution was found to be $620 \pm 70$ nm, with an asymmetry ratio of 8%. The diffraction pattern as a function of objective axial displacement is shown in Fig. 4.9.

Table 4.2 shows a summary of the results for the measured resolution, as well as what we would achieve if the optics were diffraction limited.

Table 4.2 shows that the Zeiss objective is performing at the diffraction limit for both 405 nm and 767 nm. Previous measurement made with the Zeiss objective lens [10] have found that the resolution at 767 nm is close to diffraction limited, which agrees with our data
Figure 4.8: Diffraction pattern as a function of objective’s optical axis displacement for the Zeiss objective illuminated with 405 nm light, as in Fig. 4.4. The focus was determined to correspond to the image shown in d.
Figure 4.9: Diffraction pattern as a function of objective’s optical axis displacement for the Zeiss objective illuminated with 767 nm light, as in Fig. 4.4. The focus was determined to correspond to the image shown in d.
Table 4.2: Results for measured resolution for each objective and wavelength combination. The error reported stems from an uncertainty in the magnification.

presented here; however, the previous tests showed that illumination with 405 nm gives a resolution of approximately 1.5 times the diffraction limit.

The Special Optics objective was found to have a resolution that is less than the diffraction limit for 767 nm light. This is not a physical possibility. Due to the uncertainties in the pinhole size and the sampling of the objective axial displacement, these values are supposed to represent an upper limit on the resolution. After analyzing the data as discussed above, it seems as though there are other unaccounted-for errors which cause us to overestimate the resolution in the case of the Special Optics objective at 767 nm and the Zeiss objective at 405 nm. Two obvious explanations are the errors in the fitting algorithm, specifically, determining the position of the focus for aberrated diffraction patterns, and our assumption of the incoherence of illumination.

It is therefore hard to say with certainty whether any of these values are representative of the true resolution. However, the results obtained from the tests discussed in Ch. 5 are consistent with those reported in Table 4.2 for the blue Special Optics lens, and the data reported here for the Zeiss objective illuminated with 767 nm agrees with the results given in [10]. Also, the diffraction pattern for the Zeiss objective and red light was the only combination shows very little spherical aberrations which made finding the focus of the diffraction pattern less fraught with systematic errors.
4.2.3 Resolution as a Function of Numerical Aperture

An iris is placed behind the objective in the infinity focused region of the microscope to allow us to reduce possible spherical aberrations (see Sec. 3.1.3). The iris diameter can be converted into a value of numerical aperture by

$$\text{NA} = \sin \left[ \tan^{-1} \left( \frac{D}{2\text{EFL}} \right) \right],$$  \hspace{1cm} (4.3)

where $D$ is the diameter of the aperture. The effective focal length (EFL), given by the ratio of the tube lens focal length and objective magnification, is 3.3 mm for Special Optics objective and 2.6 mm for the Zeiss. If the iris diameter is closed further than the NA of the objective, it would then be limiting the numerical aperture of the microscope and we expect the resolution to get worse, as seen by Eq. 3.3.

To measure this effect, the pinhole was focused in the center of the field of view and images were taken for different iris diameters. To minimize systematic errors, twelve different iris diameters ordered randomly.

The measured resolution as a function of iris-limited numerical aperture for the four different objective and wavelength combinations is shown in Fig. 4.10. The numerical aperture of the microscope is limited by the objective lens down to the gray vertical lines. Below that, the iris reduces the NA, and the measured resolution, here plotted as the fitted width of the Airy disk, increases as expected. This data was taken before the tube lens was positioned carefully, and therefore, the ambiguous magnification does not allow for reliable conversion of the fitted width into resolution. The point of increase in resolution, however, remains unaffected.

4.2.4 Aberrations

Another important characterization of the objective performance is the amount of aberrations, as all aberrations will act to wash out the contrast of an image. If an objective suffers from large coma or astigmatism, different point spread function shapes will be measured depending on the distance from the optical axis. This is an added complication which would have to...
be taken into account when reconstructing a lattice occupation from an image of atoms in an optical lattice. We qualitatively analyze this effect by imaging the pinhole at nine different locations, each representing a different segment of a $3 \times 3$ grid on the CCD. The field of view of the CCD is about 60 $\mu$m and thus the pinhole was translated a bit less than 30 $\mu$m in between the nine imaging locations. The data is displayed below, where each plot is spatially oriented to represent the CCD segment from which the image was taken.

The Special Optics lens shows very little to no astigmatism or coma for 405 nm light, as shown in Fig. 4.11, as there is no spatial dependence to the shape of the pinhole image. Unfortunately, as we have already seen in Fig. 4.4, this lens and wavelength combination suffers from significant spherical aberrations. This can also be seen in Fig. 4.11, as shown by the large amplitude outer rings.

When imaging with red light, the Special Optics objective shows very noticeable coma, which is seen as the asymmetric intensity in the outer rings of the diffraction pattern, Fig. 4.12. As we do not expect this objective to be diffraction limited for red light, the presence of aberrations is not a surprise; however, this indicates that our measured resolution given in Table 4.2 is much too low, as this objective is obviously not diffraction limited.

The measurements for the Zeiss objective with blue illumination, shown in Fig. 4.13, exhibits a more elliptical diffraction pattern, which may be attributed to astigmatism. Spherical aberrations are present in these measurements, and no significant dependence of the location on the transverse position of the pinhole were observed.

With red illumination, imaging with the Zeiss objective shows significant coma (Fig. 4.14); however, the coma is less strong than what is seen with the Special Optics objective and red illumination. The fact that the coma always points in one direction seems to indicate that some of the optical elements may be misaligned off axis and thus the strong coma may not be entirely a feature of the Zeiss microscope.
Figure 4.10: Fitted Airy widths as a function of the iris-limited numerical aperture. a. and b. Special Optics objective with, respectively, 405 nm and 767 nm illumination. c. and d. Zeiss objective with 405 nm and 767 nm. The gray vertical line shows where the NA is limited by the objective itself.
Figure 4.11: Diffraction patterns measured with the Special Optics objective and 405 nm illumination for nine different transverse pinhole positions with respect to the optical axis. The position of the images reflect the spatial translation of the pinhole.

Figure 4.12: Diffraction patterns measured with the Special Optics objective and 767 nm illumination, as in Fig. 4.11
Figure 4.13: Diffraction patterns measured with the Zeiss objective and 405 nm illumination, as in Fig. 4.11

Figure 4.14: Diffraction patterns measured with the Zeiss objective and 767 nm illumination, as in Fig. 4.11
Chapter 5

Nanohole Array: Resolving a 500 nm Period with Blue Imaging

The second way in which we tested the objective lenses was to image a 500 nm periodic nanohole array in transmission. Given the spacing of the holes and their small diameter of 200 nm, the transmitted light mimics that of fluorescing atoms in a lattice, and as such, these tests give us a better understanding of how well the objective will perform in the final imaging setup.

Due to the small periodicity of the array, one expects to resolve the hole pattern with only 405 nm light. This chapter investigates mostly the Special Optics objective as we did not expect the Zeiss objective to perform well enough to be able to resolve a 500 nm period [10]. It was only after the results in Ch. 4 were obtained that we attempted to image the nanohole array with the Zeiss objective. The results of the Zeiss objective tests are discussed in Sec. 5.4 and the rest of the chapter discusses results obtained with the Special Optics objective.
5.1 Silver Nanohole Array Transmission Targets

Sang-Hyun Oh and his group at the Laboratory of Nanostructures and Biosensing at the University of Minnesota fabricated five targets for our experiment (see Fig. 5.1) [19]. The targets consist of a thin silver substrate with a nanohole periodicity of 500 nm mounted on a glass slide. The silver substrate is 100 nm thick for two of the targets, and 200 nm thick for the other three. Due to the fabrication technique, the different thicknesses correspond to different hole diameters: the 100 nm thick targets have a hole size of approximately 220 nm, and the 200 nm thick targets have a hole size of approximately 120 nm.

Figure 5.1: a. Two of the silver nanohole array transmission targets. The darker region in the center of the silver substrate is the 8×8 mm area containing nanoholes. The top target was used for the majority of tests described in this report. b. and c. Scanning electron microscope photos of the nanohole array at different magnifications.

The silver substrate is mounted onto the glass slide with optical epoxy. The substrate is about 2×3 cm, and only the central 8×8 mm section contains the nanohole array. Figure 5.1a shows the region containing the holes is seen in a slightly different shade. The top target shown here, with a thickness of 100 nm, has been the most useful for the contrast measurements owing to the broken substrate. As the photo shows, there is an interface between the holed silver region and glass, which allowed us to find the focus using the edge of the target without the need of subsequently translating the target a large distance towards the final imaging region. The bottom target in Fig. 5.1a has a thickness of 200 nm. The color difference in the two targets is due to silver oxidation.
5.2 Experimental Setup and Alignment

5.2.1 Köhler illumination with a Diffuse Laser Source

As discussed in Sec. 3.1.4, one needs a large spread in $k$ vectors to wash out the diffraction pattern caused by an array of apertures. This is achieved with Köhler illumination, a common microscopy illumination technique that is designed to achieve homogeneous illumination of a target even with a structured light source. Figure 5.2 shows a basic ray diagram for Köhler illumination. The condenser lens approximately images the auxiliary lens onto the object.

![Ray diagram for Köhler illumination](image)

Figure 5.2: Ray diagram for Köhler illumination. [12]

As the Special Optics lens is sensitive to chromatic aberrations, we need a narrowband light source for illumination. However, the source must also be spatially incoherent. We create such a source by first destroying the spatial coherence of a 405 nm extended-cavity diode laser with a rotating two-diffuser system. Then we collect and reproject the light with a small aspherical lens.

Our two-diffuser system consists of one stationary diffuser placed directly in front of a rotating diffuser. A diffuser only partially destroys the spatial coherence of the laser light, leaving a speckle pattern in the imaging plane. By quickly rotating the diffuser, one can further reduce coherence and thus the size of the speckle. The addition of a second stationary diffuser allows us to achieve a similar reduction in speckle with a much slower rate of rotation [20].

A picture of the experimental setup for the Köhler illumination of the nanohole array is
shown in Fig. 5.3. The 405 nm laser beam is incident upon the pair of diffusers (Thorlabs ED1-C20) and the aspherical lens (C220TME-A) acts to direct the light towards the auxiliary lens. The auxiliary lens (LA1027A) focuses the light on the back focal plane of the large NA condenser lens (C671TME-A), and then the unfocused light is sent through the target. The iris placed behind the collector lens is imaged onto the target and can be used to adjust the field of view. Section 5.2.2 details the alignment procedure to obtain Köhler illumination with this setup.

The microscope was mounted and aligned in the same way as described in Sec. 4.1; however, for these tests, the cage system was attached to an xy translation stage. As the illumination axis is not adjustable, the microscope’s two-axis translation stage enabled us to align the microscope’s optical axis with that of the illumination. Unfortunately, this caused the system to be mechanically less stable, and images were often blurred due to the vibrations of the microscope assembly. The transmission target was mounted to an xz translation stage, which allowed for a coarse focusing and field of view selection, and the condenser lens was mounted to a z-translation stage.
5.2.2 Köhler Illumination Alignment Procedure

There are five steps to achieve Köhler illumination with the experimental setup shown in Fig. 5.3.

I. First, the objective is focused on the edge of the silver region of the target by either adjusting the microscope or the target’s translation stages. Figure 5.4a shows the silver edge imaged in this step. The white region corresponds to where light is being transmitted though the glass and saturates the CCD camera. II. The silver target is then moved aside so that light is fully transmitted through the glass slide, and the field aperture is closed until only a small amount of transmitted light is detected, Fig. 5.4b. III. In this stage, the edges of the field aperture are brought into focus by axially translating the condenser lens. Both the z-position of the condenser lens and the xy-position of the microscope are iteratively adjusted to achieve an intense, centered signal, while simultaneously the diameter of the field aperture is being decreased further. At the end of this step, the iris should be nearly closed and one should obtain an intense and evenly illuminated circle that almost fills the CCD field of view, as shown in Fig. 5.4c. IV. The target is then translated along the x-direction so that the light is transmitted through the nanohole array, Fig. 5.4d, and the field aperture is opened so its edges are just outside the field of view, Fig. 5.4e. V. The image of the nanohole array can now be finely focused with the objective’s z-translation.

The above procedure should successfully lead to Köhler illumination any time the system is originally aligned well enough that light is transmitted through the microscope to the CCD. There is no assurance that the collector lens will focus the light onto the back focal plane of the condenser lens. However, as long as the field aperture is imaged onto the target, the illumination, although maybe not truly Köhler, should be sufficiently homogeneous to achieve good images.
Figure 5.4: CCD images as seen in the five steps of the adjustment procedure to achieve Köhler illumination. The diagonal intensity modulations seen in d. and e. are caused by imperfections in the silver substrate.

5.3 Contrast Measurements

The contrast of the nanohole array images gives a measure of the resolution the microscope, as it is directly related to the width of the point spread function.

To better understand the relation between the resolution and contrast, we will model our transmission target diffraction pattern as the convolution of the diffraction limited Airy disk and an 500 nm periodic two-dimensional array of 200 nm holes. This is a simplified model because the silver is not entirely opaque to ultraviolet light and our illumination may be partially coherent. Furthermore, surface plasmons [19], which occur at metal-dielectric interfaces in subwavelength nanoholes, may affect the transmission of light. Nonetheless, the diffraction pattern for an ideal transmission target and completely incoherent illumination provides an intuitive description of the relation between image contrast and resolution, and as such, gives us an estimate for the contrasts we can expect.

For further simplicity, we will limit ourselves to a one-dimensional row of 21 holes [18]. The resulting convolution for the parameters of our transmission target, assuming a diffraction
limited system, is shown in Fig. 5.5. The horizontal axis is normalized by the hole spacing, \( d \).

![Graphs showing intensity distribution and contrast](image)

**Figure 5.5:** 
**a.** The intensity distribution for incoherent blue illumination of 21 evenly spaced 200 nm holes in a diffraction limited system with resolution \( R = 412 \) nm. 
**b.** Measured contrast as a function of the position of the point spread function’s first minimum \( r_0 \). The blue curve corresponds to Airy disks convoluted with 200 nm holes evenly separated by 500 nm. The gray curve corresponds to the contrast found from an array of Airy disks without convolution.

The contrast is defined in Eq. 3.5, where, in this discussion, the local intensity maxima and minima, \( I_{\text{max}} \) and \( I_{\text{min}} \), are taken close to the center of the diffraction pattern. For our diffracted limited system, the resulting contrast is 31.4\%, Fig. 5.5a, where \( r_0 \) is the radius to the first minimum of the Airy disk.

Figure 5.5b shows how the contrast changes as a function \( r_0/d \), where Figure 5.5a corresponds to \( r_0/d = 0.822 \). From this curve, we can convert our contrast measurements into a resolution, and as such, it is useful to create a conversion function [18], defined as

\[
C = x_0 + x_1 R + x_2 R^2 + x_3 R^3 + x_4 R^4 + x_5 R^5,
\]

where we fit a fifth order polynomial to this conversion curve for contrast values between 80 and 0.1\%. For completeness, we also look at different hole-size to hole-spacing ratios, \( D = \frac{\text{hole diameter}}{d} \), where \( D = 200/500 = 0.4 \) for our target. Table 5.1 shows the six polynomial coefficients for a number of different values of \( D \). The polynomial approximation reproduces the numerical with a deviation smaller than 1% for contrast values larger than 2\%.
Table 5.1: Polynomial coefficients for the conversion from contrast to resolution, Eq. 5.1, where \( D = 0.4 \) for our nanohole array.

<table>
<thead>
<tr>
<th>( D )</th>
<th>( x_0 )</th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_3 )</th>
<th>( x_4 )</th>
<th>( x_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.2362</td>
<td>-1.7508</td>
<td>-1.2791</td>
<td>0.46712</td>
<td>0.62134</td>
<td>-0.15576</td>
</tr>
<tr>
<td>0.2</td>
<td>1.7984</td>
<td>0.098106</td>
<td>-5.1283</td>
<td>4.7965</td>
<td>-1.8069</td>
<td>0.37312</td>
</tr>
<tr>
<td>0.4</td>
<td>1.1227</td>
<td>0.98393</td>
<td>-4.2186</td>
<td>1.8531</td>
<td>0.80136</td>
<td>-0.43919</td>
</tr>
<tr>
<td>0.5</td>
<td>1.3944</td>
<td>-1.0131</td>
<td>-1.1733</td>
<td>0.95279</td>
<td>-0.12047</td>
<td>0.046349</td>
</tr>
</tbody>
</table>

Therefore, once we get a value for contrast from our transmission target, we plug it into Eq. 5.1 with the values for \( \vec{x} \) given by the second row in Table 5.1 to obtain a value for the Rayleigh resolution of our microscope. Again, it should be mentioned that this assumes a simplified model of completely incoherent illumination of the transmission target, ignoring transmitted light through the opaque region and possible surface plasmon effects.

To get an accurate measure of the contrast, one must take into account the light transmitted through the solid silver region between the holes. This is done by measuring the number of counts per pixel in the region without holes, and subtracting this value from the image before fitting for the contrast. Without this background subtraction, one obtains a worst-case estimate as a lower limit for the contrast.

As was done with the point spread function, a number of different measurements were performed to test the quality of the microscope: determining the best-effort resolution from focused images, the contrast as a function of numerical aperture, a measure of its depth of focus, the contrast at large distances between the objective and the lens tube, and the influence of a 200 \( \mu \text{m} \) sapphire window between the target and objective.

The CCD camera settings for these tests were kept the same as for the point spread function measurements: 33 ms exposure time and zero read-out gain.
5.3.1 Algorithm to Determine Contrast

To obtain a value for the contrast from an image of the target, a sinusoidal function with a quadratic envelope was fit to the intensity distribution along one Gaussian weighted row of nanoholes. Fits are made to selected regions of interest from CCD field of view with approximate dimensions of 10×10 µm.

As the holes in the target are not perfectly aligned along the same axes as the pixels of the CCD, the coordinate system needs to be rotated first before fitting for the contrast. The relative angle between the target hole array and the CCD pixel array is determined from the Fourier transform of the image. Figure 5.6a is an image of three rows of the nanohole array with the rotated coordinate system. This allows us to choose a horizontal row of holes from which to calculate the contrast. A Gaussian weighted slice along a hand-selected row is taken to account for the pixelation of the image and to reduce noise (see Fig. 5.6b). We define our Gaussian as

\[ f = \exp\left[-\frac{(y-y_0)^2}{\sigma^2}\right]. \]

The original image is then multiplied by this weighting function and each column is summed and normalized by the area under the weighting function to get a one dimensional slice for the intensity distribution. A sinusoidal with a quadratic envelope is fit to the slice,

\[ f = [A \sin(2\pi x/d) + B] \cdot [a + bx + cx^2] \]

and the contrast is obtained from the amplitude of the sine curve as given by Eq. 3.5, or equivalently, \( C = \frac{A}{B} \). Figure 5.7 shows one such fit with a measured contrast of 13% with zero background subtraction.

![Figure 5.6](image)

**Figure 5.6:** a. After a rotation of the coordinate system, the nanohole rows appear to be horizontal. b. A Gaussian weighted slice helps to reduce the noise and account for the pixelation effect.
Figure 5.7b shows the effect of the Gaussian slice width on the contrast for one image. For small widths, the noise and pixelation make fitting difficult, and the resulting contrast is sensitive to the choice of horizontal placement of the slice. For large Gaussian widths, summing over more than one row of holes washes out the contrast. The separation between rows is approximately 4.5 pixels. A maximum in Fig. 5.7b is reached for a width of approximately 3 pixels; however, around this range, the contrast is not very sensitive to the exact choice in width, and thus a width of 3.5 pixels was used in all of the data analyses included here.

Figure 5.7: a. The amplitude obtained from the fit of a sinusoidal function with a quadratic envelope gives the contrast according to Eq. 3.5. b. The relation between measure contrast and width of Gaussian slice used in the fitting algorithm.

5.3.2 Observed Maximum Contrast and Corresponding Resolution

An important aspect of determining the contrast of an image is to perform proper background subtraction. For the initial measurements, we imaged holes in the center of the target. Then to determine the background counts, we would translate the target a few millimeters and measure the transmission through the solid silver region. Vastly different numbers for contrast were found depending on whether we imaged in the center of the target or near the edge. For a more accurate background subtraction, we began to image only near the edge of the target, so as to measure the transmission through the nanoholes and substrate without making any
adjustments to the imaging system. Figure 5.8a shows one such image, where the low-intensity region corresponds to a section of solid silver and holes are clearly visible on either side. The background was then taken as the average of counts found in the solid silver region.

Figure 5.8: a. An image near the edge of the nanohole region shows sections of solid silver (dark blue). From the pixel counts in this region, we obtain an accurate value for the background. b. Zoomed-in region of nanoholes with a contrast of 13% evaluated without background subtraction.

Even in this small field of view, Fig 5.8a, we can get contrast values that differ by almost 5% depending on which locations were fit. To account for this systematic error, multiple images are taken at multiple locations within a larger field of view. The mean contrast is found and the error on the contrast measurement is taken as the standard deviation. Figure 5.8 shows one such image where the contrast was found to be approximately 13%, with zero background subtraction, and 22%, with a background subtraction of 33 counts found from the transmission through the silver substrate.

Figure 5.9 shows some of the higher contrast images taken with our microscope. These all have contrast values between 12 and 15% with zero background subtraction, and 22 to 25% with background subtraction. A 25% contrast value corresponds to a resolution of 434 nm, according to Fig. 5.5, whereas a contrast of 15% gives a resolution of 476 nm. These resolution values here match those found for the Special Optics objective with blue illumination of the pinhole (see Sec. 4.2.2) within the reported error.

Because the number of counts is low for these pictures, a change of only two counts in the
background subtraction corresponds to a change in a couple percent in the resulting contrast. Therefore, it is difficult to say with much certainty what maximum contrast, or equivalently, what minimum resolution values could be obtained. However, the contrast values with zero background subtraction give us a trustworthy upper limit on the resolution.

5.3.3 Contrast as a Function of Objective Focus

We measured the contrast as a function of the objective’s axial position in an attempt to get an understanding of its depth of focus. The objective lens’ position was incremented by micrometer steps through the focus, and multiple images were taken at each displacement. Figure 5.10 shows the resulting contrast as a function of objective position. The error bars are the standard deviations from 15 different contrast measurements and do not include the statistical error from the fitting. The large error is a result of the way in which the microscope was mounted, causing the system to be mechanically unstable. Images often appeared to be blurred, reducing the contrast and increasing the standard deviation associated with repeated measurements.

Extracting a value for the depth of focus, as defined in Eq. 3.9, from these contrast measurements remains elusive. It is informative enough to see that the contrast remains fairly high over a range of 2-3 μm.

The contrast values shown in Fig 5.10 are fairly low compared to the highest values attained
5.3.4 Contrast as a Function of Numerical Aperture

The contrast was measured as a function of numerical aperture, as was done with the point spread function resolution in Sec. 4.2.2. Figure 5.11 shows that the contrast remains fairly constant up to just below the 0.6 numerical aperture of the objective, and then it decreases quickly as the NA of the microscope is artificially reduced. This agrees with what we saw in Sec. 4.2.3 and again indicates that an iris should be included behind the objective lens in the final microscope design.

The sloping gray line plotted for NA < 0.6 in Fig. 5.11 corresponds to the expected contrast from the model described in Fig. 5.5, assuming a diffraction limited imaging system. Our data follows the same trend as our model predicts; however, we are still able to resolve the nanoholes with an NA of 0.4. This may be an indication that our simple model for the transmission target is incomplete.
Figure 5.11: Contrast as a function of iris numerical aperture. The dotted vertical gray line at 0.6 indicates where the NA of the microscope goes from being limited by the aperture of the objective lens to being limited by the iris aperture. The light blue curve shows the expected contrast for a diffraction limited system, as found from Fig. 5.5a.

5.3.5 Large Distances between Objective and Tube Lens

The contrast was measured at numerous objective-to-tube lens distances. Figure 5.12 shows the results of each measurement. Although there seems to be a decreasing trend in contrast as the distance is increased, it is not strong and the holes could still be resolved with a distance of over 50 cm. Our current microscope stack design has this distance set as 30 cm. The data indicates that the image quality will not suffer if this distance needs to be increased by 10 cm to allow for more optics within the assembly. The error bars displayed represent the standard deviation in contrast obtained from multiple images.

As discussed in Ch. 4, an imperfect distance between tube lens and CCD alters the magnification of the microscope, and therefore, the results we found in similar tests done with the pinhole had a large error associated with them and are not presented. The change in magnification was noticeable in the contrast measurements as the sinusoidal fit frequency changed for different tube lens-to-CCD distances. Only the amplitude and not the period of the fit function affects the contrast, and as such, these measurements are not affected by the same uncertainty. However, if the distance is not optimal, the imaging plane is altered and the light exiting the objective lens is no longer infinity focused. This is expected to increase aberrations and thus
lead to a worse contrast.

### 5.4 Zeiss Objective and 405 nm Illumination of a Nanohole Array

It was only after the results obtained in Ch. 4 that we found that the nanohole array may be resolvable with the Zeiss objective. Due to time contraints, only a measure of the best-effort resolution was performed.

Figure 5.13 shows an image taken with the Zeiss objective and blue illumination. The resolution for this image was found to be about 16% with zero background subtraction and about 20% with background subtraction as described in 5.3. These results correspond to a resolution of 472 nm for zero background subtraction and 454 nm with background subtraction. There was no discernible difference in the quality of image when compared to the Special Optics objective.

### 5.5 Discussion

The Special Optics objective lens was designed specifically to work with the 200 µm sapphire vacuum window. I did not find any significant change in the contrast when the window was
An important parameter of the microscope is the working distance of the objective. Special Optics quote a working distance of 2.4 mm and I was able to measure the working distance to be $2.7 \pm 0.2$ mm with the window in place. This was done by recording the distance needed to translate the target from the front end of the objective lens until a focused image was seen. Once the target was displaced sufficiently far from the objective lens, but before the focal plane of the microscope was reached, the 200 $\mu$m sapphire window was placed in front of the objective and then the image was focused by further translation of the target. An important point to note is that the working distance can be altered by adjusting the tube lens-CCD position; however, this also affects the magnification of the microscope.

The contrast obtained with zero background subtraction gives us a reliable upper limit on the resolution of our microscope. From the images with the largest contrast and zero background subtraction, we find our microscope resolution with the Special Optics objective to be less than 480 nm, which agrees within the error of the results from the pinhole PSF measurements. The resolution of the Zeiss objective with zero background subtraction was found to be close to 470 nm.

Even without the analysis of contrast included here, the fact that we were able to resolve a 500 nm periodic array with a fair contrast with both objective lenses is extremely encouraging.
for the endeavor of imaging ultracold atoms in a 527 nm periodic optical lattice.
Chapter 6

Conclusions

We tested the resolution of two different microscope assemblies in an attempt to determine which would be better suited for initial measurements to image individual atoms in an optical lattice. The results from Ch. 4 and 5 seem to indicate that the Zeiss objective lens may be the best suited for our microscope. Unfortunately, due to the aberrations seen in the diffraction patterns, as well as possible coherence effects, one might consider additional measurements to obtain a final conclusion.

In order to eliminate the effect of coherence, one should try illuminating the pinhole with Köhler illumination; however, this is a more demanding setup due to a much lower intensity, and thus finding the pinhole with the microscope will be tedious at best. If Köhler illumination of the pinhole is achieved, and the aberrations are still prominent, one will have to find a way to minimize these before trustworthy numbers can be obtain from the fitting algorithm. Alternatively, a more complex model of the imaging system including the effect of aberrations could be developed.

Very promising results were obtained from the contrast measurements of the nanohole array. Both objective lenses were able to resolve the 500 nm period of the array with good contrast. Not only does this imply that our lattice will be resolvable with both lenses, remarkably, this demonstrates that single-site imaging can be obtained with an off-the-shelf objective
lens, greatly reducing the involved time and expenses.

The pinhole measurement indicate that the Zeiss objective with blue and red illumination and the Special Optics objective with blue illumination are nearly diffraction limited; however, strong spherical aberrations are seen for blue illumination. Partly, these aberrations could stem from off axis alignment of the microscope assembly elements and might not be solely an intrinsic property of the microscope. Therefore, to minimize these effects in the final microscope stack and in future tests, extra care should be taken to ensure all elements are placed along one common optical axis without any tilt.

The Special Optics objective was designed to be diffraction limited at 405 nm; however, the aberrations observed seem to indicate that is not the case. An investigation into the possibility of reducing the observed aberrations could be beneficial and lead further understanding of the limitations of our microscope.
Bibliography


