Quantum simulation with ultracold atoms

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CIIFAR Board Meeting

Quantum simulator?

slides @ http://ultracold.physics.utoronto.ca/talks.html
Soap films
*a simulation to find minimal surfaces.*

Problem: what is the minimal surface given fixed edges?

Answer: construct a wire grid and dip it in soap!

minimal surface for tetrahedral edges.
Answer precedes the explanation

• Lagrange: calculus of variations  
  1760: poses minimal surface problem

• Plateau: soap film simulations, c.1840

• Initiates a “Golden Age” of  
  mathematical study of minimal  
  surfaces.

• Riemann, Weierstraß, Schwarz, others:  
  fail to find answer to surfaces of least  
  area.

• Douglas: solves in 1930.  
  (Fields Medal ‘36)
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Plateau’s laws:
1. Smooth surfaces
2. Constant curvature
3. Soap films always meet in threes, and they do so at an angle of 120°, forming an edge (“Plateau Border”).
4. These Plateau Borders meet in fours at an angle of \(\arccos(-1/3)\) to form a vertex.
Soap film between two rings

Mathematical solution

Free surface:

Catenoid: zero curvature at all points

\[ r^2 = a^2 \cosh^2 \left( \frac{z}{a} \right) \]

Trapped volume:

Assumed by Plateau in 1873.

Surface proven by Hutchins et al. 2000

“Double Bubble”
Soap film between two rings

Catenoid: zero curvature at all points
Are there ‘soap bubble’ problems today?

Open questions we can answer with *simulation*?
What is simulation?

- Provides the answer to a mathematical problem or physical model
- Black box: Does not “solve” the model -- does not tell us why.
- However empirical rules might be learned; and further simulations (various initial conditions, etc) could guide understanding.

**FIRST LINE SIMULATOR:**

AND IN CASE OF FAILURE...?
Simulation of a quantum system

• Typically on a computer...a device that cannot be in a superposition or entangled state.

-Quantum Information Processing program exists because classical representation of QI inefficient.

• Feynman: “Use a quantum system to simulate another quantum system!” [1981]

Quantum Simulation (QSim)

• Not universal quantum computing...eg, couldn’t factor a number.

• Strategy: find ‘soap bubbles’: quantum systems that are natural fits for an open quantum problem.
Ultracold atoms for Quantum simulation

Program: Simulate models of quantum many-body systems including quantum materials

Questions:
“Does the Hubbard model explain d-wave superfluidity?”
“Does Stoner ferromagnetism exist?”
“Is there a quantum limit to viscosity?”
“What can resonant superfluidity tell us about high-temperature superconductivity?”
Laser system

Laser cooled atoms
Soap bubbles of the 21st century
Why use gases to simulate solids?

A: Separation of length scales

Example: Standard model physics

Atomic Physics

Quantum Chromodynamics

(string theory?)

Theories effective at each length scale.
Why use gases to simulate solids?

A: Separation of length scales

Dilute neutral gases:

100 nm
Why use gases to simulate solids?

A: Separation of length scales

Dilute neutral gases:

Interaction potential

\[ 100 \text{ nm} \]

\[ 0.5 \text{ nm} \]
Why use gases to simulate solids?

A: Separation of length scales

Dilute neutral gases:

Effectively point particles!
Where is the quantum simulation?

confinement

neutral gas

finite range

$R: \mu m$

$100 \text{ nm}$

$r_0: 1 \text{ nm}$
Where is the quantum simulation?

**SIMULATION SPACE**

1. inter-particle distance
   \[ d : 100 \text{ nm} \]

2. de Broglie wavelength:
   \[ \lambda_{\text{dB}} \lesssim d \]
   when quantum degenerate

3. scattering length:
   \[ a_s \]
   interaction strength
   \[ g = \frac{4\pi \hbar}{m} a_s \]

**confinement**

finite range

\[ R: \mu \text{m} \]

\[ r_0: 1 \text{ nm} \]
Where is the quantum simulation?

confinement

ultracold atoms

finite range

...where Hamiltonians are generic!

bosons (particles with integer spin):

\[
\hat{H} = \hat{\Psi}^\dagger \left[ -\frac{\hbar^2}{2m} \nabla^2 \right] \hat{\Psi} + \frac{g}{2} \hat{n}^2
\]

fermions (particles with half-integer spin):

\[
\hat{H} = \sum_\sigma \hat{\Psi}_\sigma^\dagger \left[ -\frac{\hbar^2}{2m} \nabla^2 \right] \hat{\Psi}_\sigma + g \hat{n}_\uparrow \hat{n}_\downarrow
\]

“Let’s simulate!”

De Broglie wavelength:

\[\lambda_{DB} \lesssim d\]

...where Hamiltonians are generic!

fermions (particles with half-integer spin):

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\]

“Let’s simulate!”
Why use gases to simulate solids?

When to quantum statistics matter?

- Helium is a superfluid when colder than $T=2K$

- Einstein’s criterion: $n\lambda_{dB}^3 \geq 1$
  (LHS of inequality scales as $\sqrt{n^2/T^3}$)

- Gas density $10^9$ times lower
  ($n\sim 10^{14}$ cm$^{-3}$ vs $10^{23}$ cm$^{-3}$)

Quantum statistics appear in gases at $T$’s a million times lower than in liquids & solids!

- In real numbers: temperatures of about 500 nK.
Question:
“What can resonant superfluidity tell us about high-temperature superconductivity?”

- Superfluidity with critical $T_c \sim 0.2\ T_F$
- In a gas: $T_F < 1\ \mu K$
- Fermi temperature in solids is $\sim 10,000\ K$
- ...so would have superconductivity at $1000\ \degree C$ if we could reproduce this physics in solids!
Question: “Quantum limit of viscosity?”

Universal Quantum Viscosity in a Unitary Fermi Gas

C. Cao, E. Elliott, J. Joseph, H. Wu, J. Petricka, T. Schäfer, J. E. Thomas

A Fermi gas of atoms with resonant interactions is predicted to obey universal hydrodynamics, in which the shear viscosity and other transport coefficients are universal functions of the density and temperature. At low temperatures, the viscosity has a universal quantum scale $\hbar n$, where $n$ is the density and $\hbar$ is Planck's constant $\hbar$ divided by $2\pi$, whereas at high temperatures the natural scale is $p_T^2/\hbar$, where $p_T$ is the thermal momentum. We used breathing mode damping to measure the shear viscosity at low temperature. At high temperature $T$, we used anisotropic expansion of the cloud to find the viscosity, which exhibits precise $\tau^{3/2}$ scaling. In both experiments, universal hydrodynamic equations including friction and heating were used to extract the viscosity. We estimate the ratio of the shear viscosity to the entropy density and compare it with that of a perfect fluid.

Ultrasound, strongly interacting Fermi gases are of broad interest because they provide a tunable tabletop paradigm for strongly interacting systems, ranging from high-temperature superconductors to nuclear matter. First observed in 2002, quantum degenerate, strong-
Tuning the simulation

Extreme Tunability of Interactions in a $^7$Li Bose-Einstein Condensate

S. E. Pollack, D. Dries, M. Junker, Y. P. Chen, T. A. Corcovilos, and R. G. Hulet

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(Received 26 November 2008; published 6 March 2009)

Trapped atom clouds:

stronger interaction “g”
Tuning the simulation

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Perspective: What can cold atoms teach us?

Quantum many-body physics is now 100 yrs old: superconductivity observed in 1911 by Kammerlingh-Onnes.

Traditional CM approach:

see phenomenon (eg, superconductivity) \rightarrow search for theory (eg, BCS model)

Quantum simulation with ultracold atoms:

quantum many-body physics (eg, BEC) \rightarrow know Hamiltonian
Quantum simulation with ultracold atoms

Conclusion:

- Ultracold atoms provide such a window of opportunity
  \[ R \gg \left\{ a_S, \lambda_{dB}, d = n^{-1/3} \approx k_F^{-1} \right\} \gg r_0 \]

- ‘Designer models’ achieved through tuning interaction strength, dynamic control, and new experimental probes

Answers to “How do we think about quantum many-body systems?”

Links to Quantum Materials, QIP, and even quantum gravity!
The H.L. Welsh Lectures in Physics 2011
University of Toronto
April 13 & 14

Deborah Jin
JILA & University of Colorado

7:00pm Wednesday, April 13
Earth Sciences Centre, ES 1050,
33 Willcocks Street
(Refreshments in Lobby afterwards)

Fun with Ultracold Atoms

Experiments with ultracold gases are among the coldest experiments in the world. I will discuss experiments where we are exploring quantum behavior in a gas of atoms cooled to temperatures near absolute zero. This talk will touch upon topics such as temperature, quantum mechanics, and superconductivity.
Thank you!