Formation of Magnetic Impurities, \( \pi \)-Junctions, and a Spontaneous Current State in a Superfluid Fermi Gas

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- **introduction**
- idea to introduce magnetic impurities to a superfluid Fermi gas
- physical properties around magnetic impurity
  - local density of states
  - localized excited (bound) states
- superfluid/ferromagnet/superfluid-junction, and spontaneous current state
1991.4~1994.3 (PhD course : Tokyo Institute of Technology)

“Antiferromagnetic spin fluctuations in high-Tc cuprates”
1991.4~1994.3 (PhD course : Tokyo Institute of Technology)

"Antiferromagnetic spin fluctuations in high-Tc cuprates"

Theory of the Thermal Conductivity of Superconducting Alloys with Paramagnetic Impurities

Vinay Ambegaokar* and Allan Griffin†
Department of Physics and Laboratory of Atomic and Solid State Physics,
Cornell University, Ithaca, New York
(Received 2 September 1964)

Fig. 6. The density of states $N_0(\omega)$ in a paramagnetic alloy function of the excitation frequency $\omega$ (we have normalized the former with respect to a normal metal and the latter with respect to the Gor’kov order parameter $\Delta$). The density of states for nonmagnetic impurities is also shown [$\omega = (r_\sigma \Delta)^{-1} = 0$].
Transition from metallic to tunneling regimes in superconducting microconstrictions: Excess current, charge imbalance, and supercurrent conversion

G. E. Blonder, M. Tinkham, and T. M. Klapwijk
Physics Department, Harvard University, Cambridge, Massachusetts 02138
(Received 19 October 1981)

and the pairs in geometries where the gap varied with position. Then, in 1971 Demers and Griffin and Griffin and Demers, calculated the transmission coefficients in N-S-N and S-N-S geometries and made the extension to a δ-function barrier at the interface. Surprisingly, this important work remained essentially unnoticed for a number of years but recently (1977) Entin-Wohlman has drawn on it to develop boundary conditions that she used in solving the Gor’kov gap equation at an N-S interface. Thus, use of the Bogoliubov equa-
1991~1994 (PhD course: Tokyo Institute of Technology)
1994.4~1995.3 (PD: Osaka University)

“Boundary effects in d-wave superconductors”

Canadian Journal of Physics

Scattering and Tunneling of Electronic Excitations in the Intermediate State of Superconductors

Jacques Demers and Allan Griffin
Department of Physics, University of Toronto, Toronto 5, Ontario
Received September 24, 1970

We give a systematic discussion of the anomalous scattering of Bogoliubov excitations at sharp N-S and S-N boundaries as well as quasi-particle tunneling in N-S-N and S-N-S geometries. The relation between the transmission resonances ($E > \Delta$) and the bound states ($E < \Delta$) in N-S-N and S-N-S geometries is explicitly exhibited. We also work out the extra thermal resistance due to scattering by a normal layer imbedded in a superconductor and by a superconducting layer imbedded in a normal metal, with results different from that obtained by Shikin.

Canadian Journal of Physics, 49, 285 (1971)
1991~1994 (PhD course : Tokyo Institute of Technology)
1994.4~1995.3 (PD: Osaka University)
1995.4~2006.3 (Tsukuba University)

“Carlson-Goldman mode and Josephson plasma in high-Tc cuprates”
1991~1994 (PhD course: Tokyo Institute of Technology)
1994.4~1995.3 (PD: Osaka University)
1995.4~2006.3 (Tsukuba University)

“Carlson-Goldman mode and Josephson plasma in high-Tc cuprates”

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Cooper-pair-condensate fluctuations and plasmons in layered superconductors

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(Received 14 January 1993)
1991~1994 (PhD course : Tokyo Institute of Technology)  
1994.4~1995.3 (PD: Osaka University)  
1995.4~2006.3 (Tsukuba University)  

2001.8~2002.6 (visiting researcher, Toronto University)  
“BCS-BEC crossover in a gas of Fermi atoms with a Feshbach resonance”

2004.9~2004.11 (visiting researcher, Toronto University)  
2006.4~ (Keio University)  
2011.5.13 and 14 (Toronto!)
Formation of Magnetic Impurities, $\pi$-Junction, and Spontaneous Current State in a Superfluid Fermi Gas

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3: CREST (JST), Japan

- Introduction
- Idea to introduce magnetic impurities to a superfluid Fermi gas
- Properties around pseudo-magnetic impurity
  - density of states
  - localized excited (bound) states
- superfluid/ferromagnet/superfluid-junction, $\pi$-junction, and spontaneous current state
Superfluid $^{40}$K and $^{6}$Li Fermi gases

“quantum simulator” to study various physical properties of superconductivity

BCS-BEC crossover by tunable interaction
- strong-coupling effects beyond mean-field theory
- pseudogap (preformed pairs)
- Fermi SF ↔ Bose SF

optical lattice
- band effect
- low-dimensional effect
- Hubbard model and strong correlation

population imbalance
- superconductivity under magnetic field
- FFLO

C. A. Regal, et al.

$|9/2, -7/2>$

$|9/2, -9/2>$
“Magnetic” effect

superfluid Fermi gas

metallic superconductivity

C. A. Regal, et al.

|9/2, −7/2⟩ → |9/2, 9/2⟩


La$_{3-x}$Gd$_x$In

c

c

magnetic impurity concentration

T_c

T_c

“magnetic” impurity

“pseudospin”

“spin”
Magnetic effects on superconductivity

Pairing mechanism

- High-Tc cuprates: antiferromagnetic spin fluctuations
- Superfluid liquid $^3$He: ferromagnetic spin fluctuations

Depairing effect

Cooper pair

Magnetic impurity
pair-breaking effect by magnetic impurities

- **suppression** of SC state by magnetic scattering
- **gapless** SC

Satori, Shiba (1992)

- **bound state** below the gap

Shiba (1968)

- competition between SC and Kondo effect

Abrikosov, Gor’kov (1961)

Nonmagnetic impurities do not affect s-wave SC. (Anderson’s theorem)

\[
\frac{T_c}{T_c^0} = \frac{\rho_{\text{imp}} \times S(S+1)u_{\text{imp}}^2 N(0)}{T_c^0} \]

\[
\frac{T_K}{\Delta} = 0.3
\]

\[
\text{Kondo singlet}
\]

\[
\text{Kondo singlet}
\]

\[
\text{impurity band}
\]

\[
\text{bound state energy / } \epsilon
\]
```
π-junction

Insulator

SC

ferromagnet

SC

Δ(x)

0

x

“0”-junction

Δ(x)

0

x

“π”-junction

Δ(x)

ferromagnet

SC

Al/Al₂O₃/PdNi

Nb

Nb

Al/Al₂O₃/PdNi
```
π-junction

Insulator

ferromagnet

“0”-junction

“π”-junction

Δ(x)

0

Δ(x)

0

SC

SC

SC

SC

Al/Al₂O₃/PdNi

Nb

Nb

Al/Al₂O₃/PdNi
We theoretically discuss an idea to introduce magnetic impurities and magnetic junction to a superfluid Fermi gas.

Phase separation in a superfluid Fermi gas with population imbalance

G. Partridge et al., PRL 97, 190407 (2006)
We theoretically discuss an idea to introduce magnetic impurities and magnetic junction to a superfluid Fermi gas.

Phase separation in a superfluid Fermi gas with population imbalance

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Phase separation in a superfluid Fermi gas with population imbalance

G. Partridge et al., PRL 97, 190407 (2006)
We theoretically discuss an idea to introduce magnetic impurities and magnetic junction to a superfluid Fermi gas.

To examine these ideas in a simple manner, we consider a Hubbard model at $T=0$. We self-consistently determine the order parameter, particle density, and polarization, around impurity potential and barrier within the mean-field level. We examine the possibilities of magnetization of impurities, SFS-, and $\pi$-junction.
two-component 2D Fermi gas \( \sigma = \uparrow, \downarrow \)

population imbalance \( N_\uparrow > N_\downarrow \)

\[
H = -t \sum_{i,j} \left[ c_{i\sigma}^\dagger c_{j\sigma} + h.c. \right] - U \sum_i n_{i\uparrow} n_{i\downarrow} + \sum_{i,\sigma} V_i n_{i\sigma} - \mu_\uparrow N_\uparrow - \mu_\downarrow N_\downarrow
\]
We solve the BdG equations to self-consistently determine $\Delta_i, \langle n_{i\sigma} \rangle, \mu_\sigma$ for given $N_\sigma$.

$$H = -t \sum_{i,j} \left[ c_{i\sigma}^\dagger c_{j\sigma} + h.c. \right] - \sum_i \Delta_i \left[ c_{i\uparrow}^\dagger c_{i\downarrow}^\dagger + h.c. \right] - U \sum_{i,\sigma} \langle n_{i\sigma} \rangle n_{i-\sigma}$$

$$+ \sum_{i,\sigma} V_i n_{i\sigma} - \mu_\uparrow N_\uparrow - \mu_\downarrow N_\downarrow + E_G$$

phase diagram of 2D uniform Hubbard model
Formation of pseudo-magnetic impurity

\[ V_i = V_0 \frac{\Gamma^2}{(R - R_{\text{imp}})^2 + \Gamma^2} \]

\[ V_0 / t = 0.25 \]
\[ \Gamma / t = 1 \]
\[ U / t = 6 \]

\[ N_{\uparrow} = 300, \; N_{\downarrow} = 300 \]
per 41×41 sites

\[ N_{\uparrow} = 301, \; N_{\downarrow} = 300 \]
per 41×41 sites

\( \Delta \) is not destroyed by impurity potential (Anderson’s theorem).

\( \Delta \) is remarkably damaged by impurity potential.
Formation of pseudo-magnetic impurity

\[ S_z = \langle n_{i\uparrow} \rangle - \langle n_{i\downarrow} \rangle \]  Local magnetization

\[ \Delta N = N_{\uparrow} - N_{\downarrow} = 0 \]

\[ \Delta N = 1 \]

\[ \Delta N = 5 \]

Nonmagnetic potential is magnetized by excess atoms!
Magnetization of nonmagnetic potential

“pseudo”-magnetic impurity

To minimize the condensation energy loss by the depairing effect, excess atoms are localized around the region where $\Delta$ is small from the beginning.

\[ H = -J\langle S_z \rangle \sigma_z \]
(classical spin)
(no exchange term)

magnetic impurity in metal

Strong electron correlation excludes double occupancy of $\uparrow$ and $\downarrow$ spins.

\[ H = -JS \cdot \sigma \]
(quantum spin)

Coulomb repulsion
We can expect that this well structure of off-diagonal pair potential $\Delta(R_x, R_y)$ induces bound states around the impurity potential, as in the case of ordinary (diagonal) potential well $V(R_x, R_y)$. 

Consider the pair-potential well $\Delta(R)$ and the potential well $V(R)$.
local density of states $\rho^s(\omega,R)$

$N^\uparrow = 301$, $N^\downarrow = 300$

In-gap states appear around "pseudo-magnetic" impurity.

uniform Fermi gas

BCS gap

$(\gamma/t = 0.05)$
The pair-potential well $\Delta(R_x, R_y)$ works as a cylindrical potential well, so that bound states can be classified by angular momentum $L$. 

$\left|\Psi_{\text{particle}}(R_x, R_y)\right|^2$
Superfluid/ferromagnet/superfluid (SFS) junction

\[ V(\mathbf{R}) \]

\[ S_z(\mathbf{R}) = \langle n_{i\uparrow} \rangle - \langle n_{i\downarrow} \rangle \]

Population imbalance

\[ P = \frac{N_{\uparrow} - N_{\downarrow}}{N_{\uparrow} + N_{\downarrow}} = 0.11 \]

\[ \frac{U}{t} = 7 \]

\[ N = 99 \]

\[ \Delta N = N_{\uparrow} - N_{\downarrow} = 11 \]
\( S_z(R) \)

\[ P = \frac{N_{\uparrow} - N_{\downarrow}}{N_{\uparrow} + N_{\downarrow}} = 0.11 \]

\( \pi \)-phase in a superfluid Fermi gas

\( \pi \)-junction

\( E/t = -875.00 \)

\( 0 \)-junction

\( E/t = -874.78 \)
The $\pi$-junction becomes stable when magnetization is large to some extent.
Effects of trap potential 2

2D cigar trap

\[ S_z(R_x, R_y) \]

\[ \Delta(R_x, R_y) \]

- \( N_\uparrow = 32 \)
- \( N_\downarrow = 28 \)
- \( U = 4t \)
- \( V_x = 0.1t \)
- \( V_y = 0.001t \)
- \( V_B = 1.0t \)
- \( \sigma = 6 \)
Because of the single-valueness of $\Delta$, the $\pi$-junction twists the phase $\theta$ of the order parameter by $\pi$ along the ring.

Spontaneous current

$$J \sim \nabla \theta > 0$$
spontaneous current state in a ring trap

\[ J \sim \nabla \theta > 0 \]

\[ N_\uparrow > N_\downarrow \]

nonmagnetic potential barrier

localized excess \(^\uparrow\)-spin atoms

\[ \pi \]-junction

\[ \Delta(x) \]

\[ \Delta = \Delta e^{\pi i} \]

1D-ring trap

\[ (N_\uparrow, N_\downarrow) = (41, 40) \]

\[ \langle \hat{n}_{i, \uparrow} \rangle, \langle \hat{n}_{i, \downarrow} \rangle \]

\[ \theta(x)/\pi \]

\[ |\Delta(x)| e^{i\theta(x)} \]

\[ \delta \theta \sim \pi \]

spontaneous flow

\[ V_{ij}/h \]

\[ R_x \]

\[ 0 \quad 0.0001 \quad 0.001 \quad 0.01 \quad 0.1 \quad 1 \quad 10 \quad 100 \quad 150 \]
We have discussed an idea to introduce magnetic impurities and magnetic junction to a superfluid Fermi gas. Using the phase separation of a polarized Fermi superfluid, we have shown that nonmagnetic potential is magnetized in the sense that some of excess atoms are localized around the potential.

**pseudo-magnetic impurities**

- polarized classical spin
- suppression of $\Delta$ around impurity

**in-gap states**

- "quantum dot"
- gapless Fermi superfluid

**superfluid/ferromagnet/superfluid junction**

- $\pi$-junction
- FFLO $\Delta_{\text{FFLO}}(x)$

spontaneous supercurrent
Summary

T. Kashimura, S. Tsuchiya, and Y. Ohashi, in preparation

pseudo-magnetic impurities

► polarized classical spin
► suppression of $\Delta$ around impurity

in-gap states
- “quantum dot”
- gapless Fermi superfluid

superfluid/ferromagnet/superfluid junction

► $\pi$-junction

$\Delta_{FFLO}(x)$

spontaneous supercurrent